Koordinat **Jurnal Pembelajaran Matematika dan Sains**
Volume 6, Number 2, July-December 2025, Pages 140-153

https://doi.org/10.24239/koordinat.v6i2.183

e-ISSN : 2745-4215

FACTORS CONTRIBUTING TO STUDENTS' DIFFICULTIES IN NUMBER PATTERN GENERALIZATION: A THEMATIC QUALITATIVE META-ANALYSIS WITHIN THE FRAMEWORK OF EARLY ALGEBRAIC THINKING

Nazwa Wulandari Azis[1], Dedi Muhtadi[1*], Sukirwan[1]

[1] Department of Mathematics Education, Siliwangi University, West Java, Indonesia.

ARTICLE INFO

Article History
Received: 29 Sep 2025
Revised: 30 Okt 2025
Accepted: 17 Nov 2025
Published: 13 Dec 2025

Keywords:
Early algebraic thinking
Mathematical difficulties
Pattern generalization
Qualitative meta-analysis
Scaffolding

ABSTRACT

This study examines students' difficulties in generalizing number patterns and their multidimensional interactions within early algebraic thinking. A thematic qualitative meta-analysis of 15 Scopus-indexed articles (2019–2024) was conducted under the PRISMA protocol using open coding and thematic clustering (κ = 0.82). Five categories of difficulty were identified: reasoning–generalization (80%), conceptual (73%), procedural–operational (60%), strategic (47%), and didactic (40%). Reasoning–generalization difficulties involved students' inability to identify relationships or formulate general rules, while conceptual and procedural weaknesses reflected misconceptions of pattern structures and errors in extending sequences. Strategic and didactic challenges were linked to limited problem-solving approaches and insufficient scaffolding. The findings highlight dynamic interactions—especially between conceptual and procedural factors—and emphasize teachers' mediating role in fostering the transition from arithmetic to algebraic symbolization.

Corresponding Author:
Dedi Muhtadi, Department of Mathematics Education, Siliwangi University, West Java, Indonesia
Email: dedimuhtadi@unsil.ac.id

How to cite:
Azis, N. W., Muhtadi, D., & Sukirwan. (2025). Factor Contributing to Student's Difficulties in Number Pattern Generalization: A Thematic Qualitative Meta-Analysis within the Framework of Early Algebraic Thinking. *Koordinat Jurnal Pembelajaran Matematika dan Sains, 6*(2), 140-153. https://doi.org/10.24239/koordinat.v6i2.183.

INTRODUCTION

The ability to generalize number patterns is a crucial foundation of early algebraic thinking, as it serves as a bridge between concrete arithmetic and abstract symbolization. According to Kieran (2022), pattern generalization enables students to grasp algebraic structures early, while Radford (2018) highlights that this process involves a cognitive transition from numerical to symbolic representations. International studies such as PISA and TIMSS confirm that many students struggle with relational and functional generalization, resulting in consistently low achievement in early algebra (OECD, 2023). The development of algebraic thinking is gradual, moving from arithmetic patterns toward symbolic expressions, with students' generalization abilities improving significantly alongside cognitive growth (Sun et al., 2023a). A focus on functional relationships particularly through recursive and covariational pattern structures has been shown to strengthen students' ability to formulate and express general rules (Pinto & Cañadas, 2021; Sterner, 2024), while the use of multiple representations concrete, numerical, graphical, and symbolic enhances students' relational reasoning and strategic flexibility (Wilkie & Hopkins, 2024;Pinto & Cañadas, 2021).

Nevertheless, understanding number patterns remains a global challenge, shaped by conceptual, procedural, strategic, and affective obstacles. González-Forte et al. (2023) found that natural number bias often impedes students' comprehension of rational numbers. (Lenz et al., 2020) highlighted that the persistent gap between conceptual and procedural knowledge reinforces recurring misconceptions a pattern also evident in students' difficulties when generalizing numerical relationships. Other difficulties arise in algebraic problem-solving contexts, including misinterpretations of word problems (Adu-Gyamfi et al., 2025), weaknesses in inverse reasoning (Cook et al., 2023), and reliance on mechanistic strategies that limit cognitive flexibility (Rudaizky et al., 2023). From an affective perspective, positive motivation has been shown to increase student engagement in generalization tasks (Kaya et al., 2022). Interventions such as the Concrete–Representational–Abstract (CRA) approach, scaffolding, and digital games promise to reduce these barriers, although their effectiveness varies across cultural and educational contexts (Kärki et al., 2022; Zwanch, 2022a). This highlights the need for adaptive instruction aligned with students' developmental stages (Blanton et al., 2024)

Moreover, the literature on students' mathematical difficulties remains fragmented regarding methodology, theoretical frameworks, and cultural contexts. Lambert & Tan (2020) noted achievement differences between students with and without disabilities, while Acosta et al. (2024) argued that the contribution of computational thinking integration to pattern generalization remains unclear. Other studies have underscored the influence of culture on students' beliefs, representations, and learning strategies (Brown et al., 2020; Dong & Kang, 2022;Star et al., 2022). Additional challenges emerge in the transition to symbolization, such as limited generalization, problem modeling errors, and functional reasoning profile variation(Sun et al., 2023; Pittalis, 2025). Early factors such as preschool number sense and arithmetic fluency significantly shape students' later algebraic achievement (Jordan et al., 2022; Psyridou et al., 2023). While these findings are valuable, no study has systematically integrated cross-study evidence to consistently and comprehensively map students' difficulties (Hunter & Miller, 2022) While these findings are valuable, the existing literature lacks a systematic integration of cross-study evidence that consistently maps students' mathematical difficulties. This methodological limitation, also reflected in (Dennis et al., 2022) through concerns about ecological and population validity, underscores the need for a more comprehensive synthesis such as the present meta-analysis.

Despite extensive research on students' difficulties in number pattern generalization, most studies have examined these challenges as isolated constructs—conceptual, procedural, or strategic—rather than as components of an interdependent cognitive system. Recent studies emphasize that students' reasoning, conceptual understanding, procedural fluency, and metacognitive control interact dynamically within early algebraic thinking ((Ellis et al., 2022; Sterner, 2024;Blanton et al., 2024). These multidimensional interactions indicate that weaknesses in one domain may trigger or amplify difficulties in another, such as when procedural errors stem from conceptual misconceptions or limited reasoning–generalization abilities. However, current research remains fragmented and lacks a systematic synthesis that explicitly explores how these difficulty dimensions co-occur and influence one another across contexts. Addressing this gap requires a comprehensive qualitative meta-analysis to reveal the interconnected nature of students' mathematical thinking difficulties.

To address this methodological and theoretical gap, the present study conducts a thematic qualitative meta-analysis of 15 Scopus-indexed international articles (2019–2024) to identify categories of students' mathematical difficulties in generalizing number patterns and to explain their multidimensional interactions. This study represents the first qualitative meta-analysis to explicitly map conceptual, procedural, reasoning–generalization, strategic, and didactic difficulties within the framework of early algebraic thinking, while offering a novel contribution by positioning teacher instruction as a key mediating variable in students' transition from concrete arithmetic to algebraic symbolization.

METHOD

Thematic qualitative meta-analysis, as outlined by Thomas & Harden (2008), has proven effective in capturing contextual meanings and identifying cross-study patterns, making it highly suitable for mapping students' difficulties with number patterns through recurring categories (Lochmiller, 2021). This approach enables systematic mapping of students' problems and their connections with instructional factors, unlike descriptive methods (Sandelowski et al., 2020) or interpretive methods such as meta-ethnography (Kakos & Fritzsche, 2017). Within educational research, this method is beneficial for examining trends across studies while providing in-depth insights into instructional design (Leary & Walker, 2018)

Literature Search Strategy

The literature search was conducted systematically through Scopus, Web of Science, ERIC, and Google Scholar for publications from 2019 to 2024, using Boolean operators (AND, OR). This process initially yielded 136 articles, then narrowed primary studies based on methodological and substantive criteria (Bramer et al., 2017; Aba-Oli et al., 2025; van Wee & Banister, 2024). The screening procedure followed the PRISMA 2020 protocol to ensure transparency, reproducibility, and credibility of findings, although challenges such as database syntax limitations and the absence of dedicated databases remained (Bramer et al., 2017; Aba-Oli et al., 2025; van Wee & Banister, 2024). The article identification and selection stages are presented in Table 1, adapted from the PRISMA 2020 flowchart, to visually clarify the inclusion process.

Table 1. PRISMA 2020 Flow Diagram for Article Selection

Stage	Description	Number of Articles
Identification	Articles identified through database searching (Scopus, Web of Science, ERIC, Google Scholar)	136
	Duplicates removed	24
Screening	Titles and abstracts screened based on relevance to mathematical pattern generalization	112
	Articles excluded after abstract screening	62

Eligibility	Full-text articles assessed for methodological and thematic criteria	50
	Articles excluded (quantitative-only, non-peer-reviewed, or outside topic scope)	35
Inclusion	Qualitative or mixed-method studies meeting all inclusion criteria	15

Inclusion and Exclusion Criteria

Strict inclusion and exclusion criteria were applied to ensure relevance and quality. Eligible studies included qualitative or mixed-methods research with primary data from elementary to secondary school students, published in peer-reviewed international journals indexed in Scopus or the Web of Science between 2019 and 2024 (Baumanns et al., 2024; Junker et al., 2025). Methodological quality was assessed using the CASP Checklist, while conceptual papers, purely quantitative studies, grey literature, and articles with minimal data were excluded (Lariviere et al., 2025; Parey & Kutscher, 2024; Reid O'Connor, 2024). Although this approach may have excluded some innovative insights, it ensured a focus on rigorous and valid studies. The inclusion and exclusion criteria applied in selecting the reviewed studies are summarized in Table 2.

Table 2. Inclusion and Exclusion Criteria Used in the Meta-Analysis

Aspect	Inclusion Criteria	Exclusion Criteria
Type of study	Qualitative or mixed-methods with primary data	Conceptual, purely quantitative, grey literature
Subjects	Elementary–secondary students	Teachers, university students, or non-student samples
Sources	International journals indexed in Scopus/WoS	Non-indexed articles or unindexed conference proceedings
Publication period	2019–2024	Before 2019
Methodological quality	Passed CASP checklist quality assessment	Articles with very limited or non-transparent data

Fifteen qualitative studies that met the inclusion criteria were analyzed thematically. These studies, summarized in Table 3, collectively explore students' conceptual, procedural, reasoning–generalization, strategic, and didactic difficulties within the framework of early algebraic thinking.

Table 3. Summary of Reviewed Studies on Early Algebraic Thinking

Author (s) and Year	Title of the Article	Journal Name
Kieran (2022)	The Multi-dimensionality of Early Algebraic Thinking: Background, Overarching Dimensions, and New Directions	ZDM – Mathematics Education
Pinto & Cañadas (2021)	Generalizations of Third and Fifth-Graders within a Functional Approach to Early Algebra	Mathematics Education Research Journal
Ellis et al. (2022)	Generalization Across Multiple Mathematical Domains: Relating, Forming, and Extending	Cognition and Instruction
Junker et al. (2025)	Patterning Strategies in Grade 1 Students with Low and High Number Sense Proficiency	Educational Studies in Mathematics
González-Forte et al. (2023)	Incorrect Ways of Thinking About the Size of Fractions	International Journal of Science and Mathematics Education

Hunter & Miller (2022)	Using a Culturally Responsive Approach to Develop Early Algebraic Reasoning with Young Diverse Learners	International Journal of Science and Mathematics Education
Zwanch (2022)	Using Number Sequences to Account for Differences in Generalizations	School Science and Mathematics
Sterner (2024)	Using Graphical Representations to Develop Students' Covariational Thinking in Pattern Generalizations	International Journal of Science and Mathematics Education
Sun et al. (2023)	The Developmental Progression of Early Algebraic Thinking of Elementary School Students	Journal of Intelligence
Ramos-Franco & Aké (2024)	Visual Supports in Students' Pattern Generalization Tasks	Educational Studies in Mathematics
Lee (2025)	Concept-Focused and Procedure-Focused Instruction on Algebra Performance	Journal of Learning Disabilities
Kim et al. (2023)	A Multilevel Meta-Analysis of Whole Number Computation Interventions for Students with Learning Difficulties	Remedial and Special Education
Torres et al. (2024)	Identifying Misconceptions in Functional Thinking	Journal of Mathematical Behavior
Wang & Lewis (2020)	Analytical Chemistry Students' Explanatory Statements in Relation to Lecture Context	Chemistry Education Research and Practice
Blanton et al. (2024)	Progressions in Young Learners' Understanding of Parity Arguments	Mathematical Thinking and Learning

Data Analysis Procedure

Data analysis followed Thomas and Harden's framework, including open coding, subtheme grouping, and thematic synthesis to identify mathematical thinking difficulties. NVivo 12 was used to maintain consistency and provide an audit trail, with reliability tested using Cohen's kappa (κ = 0.82) (Belotto, 2018; Melhuish et al., 2020). Beyond ensuring objectivity and transparency, this approach also allows integration with quantitative methods to generate more comprehensive analyses (Guetterman & Fetters, 2018; Czocher & Melhuish, 2024).

Validity, Credibility, and Rigor

Validity, credibility, and rigor were maintained through Lincoln and Guba's (1985) strategies, including triangulation, audit trails with NVivo, tracing themes back to raw data, and contextual descriptions to support transferability (Morse, 2015; Amin et al., 2020; Byram, 2022; Spiers et al., 2018; Janis, 2022).Although debates persist over the terminology of validity and reliability (Morse, 2015; Spiers et al., 2018), these verification strategies reinforced methodological rigor and the trustworthiness of findings.

Ethical Considerations

Ethical considerations focused on accurate citation, acknowledgment of original authors, and adherence to academic standards. Since this literature-based study did not involve human participants, formal ethical approval was not required (Blažun Vošner et al., 2017; Yıldırım et al., 2019; Mehta et al., 2023; Thurtle et al., 2021). Nevertheless, academic integrity was safeguarded through consistent application of citation styles (APA, MLA, Chicago) and compliance with publication ethics (Blažun Vošner et al., 2017; Mehta et al., 2023).

Reporting Transparency

Finally, transparency in reporting was prioritized to ensure that findings are reliable, reproducible, and widely accessible. Documentation included article characteristics (title, author, year, context, method), detailed records of excluded articles with reasons, and comprehensive reporting of data, analytic scripts, and missing data (Winker et al., 2023; Teare, 2016; Pulverer, 2014). Open practices such as preregistration, data sharing, and adoption of open science badges are increasingly emphasized (Kerig, 2020; Lopez et al., 2024). Nevertheless, their implementation remains limited and challenging in low- and middle-income countries (Winker et al., 2023).

RESULT AND DISCUSSION

The meta-analysis of 15 qualitative articles identified five main categories of mathematical thinking difficulties in number pattern generalization: (1) conceptual, (2) procedural–operational, (3) reasoning–generalization, (4) strategic, and (5) didactic. The frequency distribution shows that reasoning–generalization (80%) and conceptual (73%) difficulties are the most dominant, while procedural–operational (60%), strategic (47%), and didactic (40%) difficulties appear less frequently but remain significant as reinforcing factors.

These percentages represent the proportion of the 15 reviewed studies in which each category of difficulty was reported. Because a single study could include more than one type of difficulty, the total exceeds 100%. Therefore, the percentages reflect the occurrence rate of each difficulty category across studies, not a cumulative total. difficulties appear less frequently but remain significant as reinforcing factors.

The frequency analysis across the 15 reviewed studies revealed five major categories of students' difficulties in generalizing number patterns: reasoning–generalization, conceptual, procedural–operational, strategic, and didactic.Among these, reasoning–generalization difficulties were reported most frequently (80%), followed by conceptual difficulties (73%). Procedural–operational (60%), strategic (47%), and didactic (40%) difficulties occurred less often but were still significant as reinforcing factors.

These percentages represent the proportion of studies in which each type of difficulty appeared. Because one study could report more than one category, the total exceeds 100%. Therefore, the values should be interpreted as the occurrence rate of each category across studies rather than a cumulative total. This textual presentation replaces the graphical form to maintain reporting clarity and avoid redundancy.

Reasoning–generalization difficulties were reported in 12 studies, conceptual in 11, procedural–operational in 9, strategic in 7, and didactic in 6. These frequencies do not indicate separate or mutually exclusive groups of studies. Since each of the 15 reviewed articles could report more than one type of difficulty, several categories overlap. Therefore, the total number across categories exceeds 15, even though the analysis was based on the same set of 15 qualitative studies. This distribution underscores the dominance of reasoning–generalization and conceptual difficulties, with the other categories often emerging as consequences or amplifiers.

Conceptual Difficulties

Conceptual difficulties were consistently found in 11 studies, characterized by students' failure to understand the structural regularity of patterns, connect relationships between terms, or extract the underlying numerical principles of a general rule. Many students tended to overgeneralize from only a few initial terms without testing the consistency of the pattern. These difficulties were more prominent at the middle school level, when formal symbolism is first introduced, aligning with Bruner's theory of representation, emphasizing that the transition from enactive and iconic to symbolic stages requires systematic instructional support.

Procedural–Operational Difficulties

Procedural–operational difficulties were reported in 9 studies, including incorrect substitutions, arithmetic errors, or inappropriate application of formulas. These difficulties rarely stood alone but were typically rooted in conceptual misconceptions. For example, students who failed to understand the recursive nature of a pattern were more prone to making substitution errors. This finding reinforces that procedural practice without conceptual understanding tends to strengthen misconceptions.

Reasoning–Generalization Difficulties

Reasoning–generalization difficulties were the most dominant, reported in 12 studies. These included challenges in formulating conjectures, testing pattern validity, and symbolizing generalizations. Many students remained at the concrete–operational stage, able to recognize numerical regularities but unable to extract relational principles. This aligns with Ellis et al. (2022) findings that students often become "trapped" at a surface pattern level. Some studies, such as Ramos-Franco and Aké (2024), indicated that visual supports can mitigate these difficulties, underscoring that the ability to generalize is strongly influenced by instructional quality.

Strategic Difficulties

Strategic difficulties were identified in 7 studies, reflected in the use of unproductive problem-solving strategies. Students often applied recursive strategies mechanistically without deriving explicit forms or used additive approaches in multiplicative patterns. While these activities appeared active, they revealed entrenchment in familiar procedures and limited cognitive flexibility.

Didactic Difficulties

Didactic difficulties were found in 6 studies, including overly procedural instruction, insufficient scaffolding, and misconceptions among pre-service teachers. These factors often compound conceptual and generalization difficulties. However, research such as Sibgatullin et al. (2022) demonstrated that technology-based representational tools can facilitate students' transition to symbolic thinking, highlighting the dual role of instructional design quality as both a potential obstacle and a solution.

The analysis of frequency distribution revealed that students' mathematical thinking difficulties in number pattern generalization were categorized into five dimensions: reasoning–generalization, conceptual, procedural–operational, strategic, and didactic. Among these, reasoning–generalization difficulties appeared in 80% of the reviewed studies, making them the most dominant. Conceptual difficulties followed with 73%, while procedural–operational difficulties were identified in 60% of the studies. Strategic difficulties occurred in 47% and didactic difficulties in 40%.

These percentages represent the proportion of the 15 reviewed studies in which each category of difficulty was reported. Because a single study could include more than one difficulty type, the total exceeds 100%. Therefore, the percentages should be interpreted as the occurrence rate of each difficulty category across studies rather than a cumulative total. This textual presentation replaces the graphical representation to maintain reporting clarity and avoid redundancy.

DISCUSSION

Dominant Difficulties in Pattern Generalization

The meta-analysis of 15 qualitative studies revealed that reasoning–generalization and conceptual difficulties constitute the primary challenges faced by students in number pattern generalization. Reasoning–generalization difficulties often arise when students recognize only surface-level numerical regularities without extracting the underlying relational principles (Ellis et al., 2024; Zwanch, 2022; De Smedt, 2022). Conceptual difficulties, such as misconceptions of recursive structures, frequently serve as the root of procedural errors (Nzemeke, 2023).These findings highlight that the development of early algebraic thinking is inseparable from the integration of conceptual understanding and the ability to generalize symbolically (Sun et al., 2023; Torres et al., 2024).

Multidimensional Interactions among Factors

The analysis demonstrated dynamic interactions among reasoning–generalization, conceptual, procedural–operational, strategic, and didactic difficulties. Procedural difficulties typically stem from conceptual misconceptions, while mechanistic strategies devoid of relational understanding exacerbate strategic challenges (Kieran, 2022), pattern generalization. Procedural drills without conceptual grounding reinforce misconceptions, whereas conceptually informed procedural skills enhancer accuracy (Scheibling-Sève et al., 2020). Didactic factors also play a dual role:

procedural instruction and teacher misconceptions aggravate students' difficulties, but instructional designs emphasizing multiple representations and interactive technologies can mitigate them (Kaya et al., 2022).Thus, mathematics learning should be viewed as an interactive system linking cognitive, strategic, and didactic dimensions.

To make these interactions clearer, Table 4 summarizes the interconnections among the five dimensions of mathematical difficulties identified in this study.

Table 4. Summary of Interconnections among Dimensions of Mathematical Difficulties

Dimension	Interconnected With	Nature of Interaction
Reasoning Generalization and Conceptual	Procedural Operational	Conceptual misconceptions and limited reasoning skills hinder the formulation of general rules, while weak procedural fluency reinforces surface-level pattern recognition..
Procedural Operational and Strategic	Conceptual, Didactic	Incomplete conceptual understanding leads to procedural errors and unproductive strategies, often amplified by teacher-centered instruction.
Didactic (Instructional Factors)	All other dimensions	Didactic limitations such as minimal scaffolding or lack of multiple representations exacerbate conceptual and procedural difficulties but can also serve as mediating solutions through adaptive instruction.

Instructional Implications and Theoretical Perspectives

These findings extend Bruner's framework of representational transition (enactive–iconic–symbolic) by emphasizing staged scaffolding as essential for supporting students' representational shifts. In line with Vygotsky's zone of proximal development, teacher instruction is a crucial mediating variable for overcoming conceptual and procedural limitations (Margolis, 2020; Thompson, 2023). From a cross-cultural perspective, pattern generalization difficulties appear universal, but intervention strategies must be context-sensitive. While digital technologies have proven effective in developed contexts (Gennen, 2023; Chen et al., 2023), their efficacy in developing countries is powerfully shaped by educational infrastructure (Rodriguez et al., 2024). Consequently, cross-national generalizations should account for sociocultural and resource-based differences.

Problem-Solving Strategies and Cognitive Flexibility

Seven studies reported that students often relied on problem-solving strategies that were superficially productive but essentially mechanistic and limited. They used recursive or additive approaches without deriving explicit forms or generative strategies(Rea et al., 2022;Durkin et al., 2023;Brod, 2021). This reliance reduced cognitive flexibility. Interventions emphasizing strategic reasoning have been found to improve students' cognitive flexibility.

Comparative analysis, the derivation of explicit formulas, and metacognitive reflection have been shown to enhance students' strategic reasoning (Fabry & Pantsar, 2021;Adu-Gyamfi et al., 2025).

The Role of Didactic Factors

Teacher-centered procedural instruction, teacher misconceptions, and curricular pressures often intensified students' difficulties (Lee, 2025;Kim et al., 2023; Rojo et al., 2023;Rojo et a). Yet didactic factors may also serve as solutions when appropriately designed. Multiple representations, interactive technologies, and activity-based learning approaches have been shown to support the transition from concrete arithmetic to symbolic reasoning (Post & Prediger, 2024;Wang & Lewis, 2020;Rexigel et al., 2024). With thoughtful instructional design, didactic challenges can be transformed into opportunities for more inclusive and practical learning.

Limitations and Directions for Future Research

Despite providing a comprehensive synthesis, this meta-analysis has several limitations. First, its scope was limited to Scopus-indexed articles from 2019–2024, which raises the possibility of publication bias. Second, not all primary studies provided rich qualitative data, resulting in varying levels of depth across studies. Third, cultural differences among studies complicate broad generalizations. Future research should: (a) integrate qualitative and quantitative meta-analyses to produce more holistic insights, (b) conduct cross-cultural examinations of instructional practices in both developed and developing countries, and (c) undertake longitudinal studies to test the sustained effectiveness of scaffolding, multiple representations, and digital technologies.

Scientific Contribution and Novelty

The primary contribution of this study lies in its mapping of multidimensional interactions among conceptual, procedural, reasoning–generalization, strategic, and didactic difficulties—an approach rarely examined simultaneously in prior research. By positioning teacher instruction as a key mediating variable, the study extends the framework of early algebraic thinking from focusing on isolated difficulties to an integrated system-level understanding. The cognitive, strategic, and didactic perspectives within a multidimensional interactive framework, thereby offering new directions for designing mathematics instruction that is holistic, adaptive, and culturally responsive.

CONCLUSION

This study represents the first thematic qualitative meta-analysis to systematically map students' mathematical thinking difficulties in number pattern generalization into five major categories: conceptual, procedural, reasoning–generalization, strategic, and didactic. The findings confirm that reasoning–generalization and conceptual difficulties are the most dominant, while procedural–operational, strategic, and didactic difficulties function as reinforcing factors that dynamically interact with them. The novelty of this study lies in its identification of multidimensional interactions among difficulties and its emphasis on teacher instruction as a key mediating variable within the framework of early algebraic thinking. Accordingly, this research extends the field's understanding from merely mapping difficulties to a more comprehensive, adaptive, and context-sensitive interactive framework.

AUTHOR CONTRIBUTIONS

First Author : Conceptualization, investigation, formal analysis, data curation, writing – original draft, and visualization.

Second Author and Third Author : Writing – review & editing, validation, methodology, and supervision.

REFERENCES

Aba-Oli, Z., Koyas, K., & Husen, A. (2025). Higher-order thinking skills-oriented problem-based learning interventions in mathematics: A systematic literature review. *School Science and Mathematics*, *125*(3), 214–231. https://doi.org/10.1111/ssm.12676

Acosta, Y., Alsina, Á., & Pincheira, N. (2024). Computational thinking and repetition patterns in early childhood education: Longitudinal analysis of representation and justification. *Education and Information Technologies*, *29*(6), 7633–7658. https://doi.org/10.1007/s10639-023-12051-6

Adu-Gyamfi, K., Chandler, K., & Thompson, A. (2025a). Algebra story problem: The nature of students' obstacles. *School Science and Mathematics*, *125*(2), 140–153. https://doi.org/10.1111/ssm.12659

Adu-Gyamfi, K., Chandler, K., & Thompson, A. (2025b). Algebra story problem: The nature of students' obstacles. *School Science and Mathematics*, *125*(2), 140–153. https://doi.org/10.1111/ssm.12659

Amin, M. E. K., Nørgaard, L. S., Cavaco, A. M., Witry, M. J., Hillman, L., Cernasev, A., & Desselle, S. P. (2020). Establishing trustworthiness and authenticity in qualitative pharmacy research. *Research in Social and Administrative Pharmacy*, *16*(10), 1472–1482. https://doi.org/10.1016/j.sapharm.2020.02.005

Baumanns, L., Pitta-Pantazi, D., Demosthenous, E., Lilienthal, A. J., Christou, C., & Schindler, M. (2024). Pattern-Recognition Processes of First-Grade Students: An Explorative Eye-Tracking Study. *International Journal of Science and Mathematics Education*, *22*(8), 1663–1682. https://doi.org/10.1007/s10763-024-10441-x

Belotto, M. (2018). Data Analysis Methods for Qualitative Research: Managing the Challenges of Coding, Interrater Reliability, and Thematic Analysis. *The Qualitative Report*. https://doi.org/10.46743/2160-3715/2018.3492

Blanton, M., Gardiner, A. M., Ristroph, I., Stephens, A., Knuth, E., & Stroud, R. (2024). Progressions in young learners' understandings of parity arguments. *Mathematical Thinking and Learning*, *26*(1), 90–121. https://doi.org/10.1080/10986065.2022.2053775

Blažun Vošner, H., Železnik, D., Kokol, P., Vošner, J., & Završnik, J. (2017). Trends in nursing ethics research: Mapping the literature production. *Nursing Ethics*, *24*(8), 892–907. https://doi.org/10.1177/0969733016654314

Bramer, W. M., Rethlefsen, M. L., Kleijnen, J., & Franco, O. H. (2017). Optimal database combinations for literature searches in systematic reviews: a prospective exploratory study. *Systematic Reviews*, *6*(1), 245. https://doi.org/10.1186/s13643-017-0644-y

Brod, G. (2021). Generative Learning: Which Strategies for What Age? *Educational Psychology Review*, *33*(4), 1295–1318. https://doi.org/10.1007/s10648-020-09571-9

Brown, J. L., Ortiz-Padilla, M., & Soto-Varela, R. (2020). Does Mathematical Anxiety Differ Cross-Culturally? *Journal of New Approaches in Educational Research*, *9*(1), 126–136. https://doi.org/10.7821/naer.2020.1.464

Byram, J. (2022). Do you trust me? Rigor and Information Power in Qualitative Research. *The FASEB Journal*, *36*(S1). https://doi.org/10.1096/fasebj.2022.36.S1.0I620

Cook, J. P., Richardson, A., Strand, S., Reed, Z., & Melhuish, K. (2023). Examining the concept of inverse: Theory-building via a standalone literature review. *The Journal of Mathematical Behavior*, *72*, 101100. https://doi.org/10.1016/j.jmathb.2023.101100

Czocher, J. A., & Melhuish, K. (2024). Attending to Coherence Among Research Questions, Methods, and Claims in Coding Studies. *Journal for Research in Mathematics Education*, *55*(3), 148–155. https://doi.org/10.5951/jresematheduc-2022-0037

De Smedt, B. (2022). Individual differences in mathematical cognition: a Bert's eye view. *Current Opinion in Behavioral Sciences*, *46*, 101175. https://doi.org/10.1016/j.cobeha.2022.101175

Dennis, M. S., Sorrells, A. M., Chovanes, J., & Kiru, E. W. (2022). Ecological and Population Validity of Mathematics Interventions for Diverse Students With Low Mathematics Achievement: A Meta-Analysis. *Learning Disability Quarterly*, *45*(1), 32–45. https://doi.org/10.1177/07319487211010342

Dong, L., & Kang, Y. (2022). Cultural differences in mindset and beliefs regarding mathematics learning. *Current Opinion in Behavioral Sciences*, *46*, 101159. https://doi.org/10.1016/j.cobeha.2022.101159

Durkin, K., Rittle-Johnson, B., Star, J. R., & Loehr, A. (2023). Comparing and Discussing Multiple Strategies: An Approach to Improving Algebra Instruction. *The Journal of Experimental Education*, *91*(1), 1–19. https://doi.org/10.1080/00220973.2021.1903377

Ellis, A. B., Lockwood, E., Tillema, E., & Moore, K. (2022). Generalization Across Multiple Mathematical Domains: Relating, Forming, and Extending. *Cognition and Instruction*, *40*(3), 351–384. https://doi.org/10.1080/07370008.2021.2000989

Ellis, A. B., Waswa, A., Tasova, H. I., Hamilton, M., Moore, K. C., & Çelik, A. (2024). Classroom Supports for Generalizing. *Journal for Research in Mathematics Education*, *55*(1), 7–30. https://doi.org/10.5951/jresematheduc-2022-0140

Fabry, R. E., & Pantsar, M. (2021). A fresh look at research strategies in computational cognitive science: The case of enculturated mathematical problem solving. *Synthese*, *198*(4), 3221–3263. https://doi.org/10.1007/s11229-019-02276-9

González-Forte, J. M., Fernández, C., Van Hoof, J., & Van Dooren, W. (2023). Incorrect Ways of Thinking About the Size of Fractions. *International Journal of Science and Mathematics Education*, *21*(7), 2005–2025. https://doi.org/10.1007/s10763-022-10338-7

Guetterman, T. C., & Fetters, M. D. (2018). Two Methodological Approaches to Integrating Mixed Methods and Case Study Designs: A Systematic Review. *American Behavioral Scientist*, *62*(7), 900–918. https://doi.org/10.1177/0002764218772641

Hunter, J., & Miller, J. (2022). Using a Culturally Responsive Approach to Develop Early Algebraic Reasoning with Young Diverse Learners. *International Journal of Science and Mathematics Education*, *20*(1), 111–131. https://doi.org/10.1007/s10763-020-10135-0

Janis, I. (2022). Strategies for Establishing Dependability between Two Qualitative Intrinsic Case Studies: A Reflexive Thematic Analysis. *Field Methods*, *34*(3), 240–255. https://doi.org/10.1177/1525822X211069636

Jordan, N. C., Devlin, B. L., & Botello, M. (2022). Core foundations of early mathematics: refining the number sense framework. *Current Opinion in Behavioral Sciences*, *46*, 101181. https://doi.org/10.1016/j.cobeha.2022.101181

Junker, A., Nortvedt, G. A., & Farsani, D. (2025). Patterning strategies in grade 1 students with low and high number sense proficiency. *Educational Studies in Mathematics*, *118*(1), 29–51. https://doi.org/10.1007/s10649-024-10341-5

Kakos, M., & Fritzsche, B. (2017). Meta-ethnography E& E. *Ethnography and Education*, *12*(2), 129–133. https://doi.org/10.1080/17457823.2017.1278578

Kärki, T., McMullen, J., & Lehtinen, E. (2022). Improving rational number knowledge using the NanoRoboMath digital game. *Educational Studies in Mathematics*, *110*(1), 101–123. https://doi.org/10.1007/s10649-021-10120-6

Kaya, Z., Kaya, O. N., Aydemir, S., & Ebenezer, J. (2022). Knowledge of Student Learning Difficulties as a Plausible Conceptual Change Pathway Between Content Knowledge and Pedagogical Content Knowledge. *Research in Science Education*, *52*(2), 691–723. https://doi.org/10.1007/s11165-020-09971-5

Kerig, P. K. (2020). Open Science Practices at the *Journal of Traumatic Stress*. *Journal of Traumatic Stress*, *33*(2), 133–136. https://doi.org/10.1002/jts.22489

Kieran, C. (2022). The multi-dimensionality of early algebraic thinking: background, overarching dimensions, and new directions. *ZDM – Mathematics Education*, *54*(6), 1131–1150. https://doi.org/10.1007/s11858-022-01435-6

Kim, S. A., Bryant, D. P., Bryant, B. R., Shin, M., & Ok, M. W. (2023). A Multilevel Meta-Analysis of Whole Number Computation Interventions for Students With Learning Disabilities. *Remedial and Special Education*, *44*(4), 332–347. https://doi.org/10.1177/07419325221117293

Lambert, R., & Tan, P. (2020). Does disability matter in mathematics educational research? A critical comparison of research on students with and without disabilities. *Mathematics Education Research Journal*, *32*(1), 5–35. https://doi.org/10.1007/s13394-019-00299-6

Lariviere, D. O., Arsenault, T. L., & Payne, S. B. (2025). A literature review: Mathematics vocabulary intervention for students with mathematics difficulty. *School Science and Mathematics*, *125*(1), 6–17. https://doi.org/10.1111/ssm.12684

Leary, H., & Walker, A. (2018). Meta-Analysis and Meta-Synthesis Methodologies: Rigorously Piecing Together Research. *TechTrends*, *62*(5), 525–534. https://doi.org/10.1007/s11528-018-0312-7

Lee, J. (2025). Concept-Focused and Procedure-Focused Instruction on the Algebra Performance of Grade 9 Students With and Without Mathematics Difficulty. *Journal of Learning Disabilities*, *58*(3), 192–209. https://doi.org/10.1177/00222194241249960

Lenz, K., Dreher, A., Holzäpfel, L., & Wittmann, G. (2020). Are conceptual knowledge and procedural knowledge empirically separable? The case of fractions. *British Journal of Educational Psychology*, *90*(3), 809–829. https://doi.org/10.1111/bjep.12333

Lochmiller, C. (2021). Conducting Thematic Analysis with Qualitative Data. *The Qualitative Report*. https://doi.org/10.46743/2160-3715/2021.5008

Lopez, D. A., Cardenas-Iniguez, C., Subramaniam, P., Adise, S., Bottenhorn, K. L., Badilla, P., Mukwekwerere, E., Tally, L., Ahanmisi, O., Bedichek, I. L., Matera, S. D., Perez-Tamayo, G. M., Sissons, N., Winters, O., Harkness, A., Nakiyingi, E., Encizo, J., Xiang, Z., Wilson, I. G., … Huber, R. S. (2024). Transparency and reproducibility in the Adolescent Brain Cognitive Development (ABCD) study. *Developmental Cognitive Neuroscience*, *68*, 101408. https://doi.org/10.1016/j.dcn.2024.101408

Margolis, A. A. (2020). Zone of Proximal Development, Scaffolding and Teaching Practice. *Cultural-Historical Psychology*, *16*(3), 15–26. https://doi.org/10.17759/chp.2020160303

Mehta, P., Zimba, O., Gasparyan, A. Y., Seiil, B., & Yessirkepov, M. (2023). Ethics Committees: Structure, Roles, and Issues. *Journal of Korean Medical Science*, *38*(25). https://doi.org/10.3346/jkms.2023.38.e198

Melhuish, K., Thanheiser, E., & Guyot, L. (2020). Elementary school teachers notice essential mathematical reasoning forms: justification and generalization. *Journal of Mathematics Teacher Education*, *23*(1), 35–67. https://doi.org/10.1007/s10857-018-9408-4

Morse, J. M. (2015). Critical Analysis of Strategies for Determining Rigor in Qualitative Inquiry. *Qualitative Health Research*, *25*(9), 1212–1222. https://doi.org/10.1177/1049732315588501

OECD. (2023). *PISA 2022 Results (Volume I)*. OECD. https://doi.org/10.1787/53f23881-en

Parey, B., & Kutscher, E. (2024). Operationalizing and Expanding the Rigorous Mixed Methods Framework for Systematic Reviews: The Case of Inclusive Education. *Journal of Mixed Methods Research*, *18*(2), 147–164. https://doi.org/10.1177/15586898231175192

Pinto, E., & Cañadas, M. C. (2021a). Generalizations of third and fifth-graders within a functional approach to early algebra. *Mathematics Education Research Journal*, *33*(1), 113–134. https://doi.org/10.1007/s13394-019-00300-2

Pittalis, M. (2025). An empirically validated rational number sense framework. *Mathematics Education Research Journal*, *37*(1), 125–160. https://doi.org/10.1007/s13394-024-00484-2

Post, M., & Prediger, S. (2024). Teaching practices for unfolding information and connecting multiple representations: the case of conditional probability information. *Mathematics Education Research Journal*, *36*(1), 97–129. https://doi.org/10.1007/s13394-022-00431-z

Psyridou, M., Torppa, M., Tolvanen, A., Poikkeus, A.-M., Lerkkanen, M.-K., & Koponen, T. (2023). Developmental profiles of arithmetic fluency skills from grades 1 to 9 and their early identification. *Developmental Psychology*, *59*(12), 2379–2396. https://doi.org/10.1037/dev0001622

Pulverer, B. (2014). Transparent, Reproducible Data. *The EMBO Journal*, *33*(22), 2597–2597. https://doi.org/10.15252/embj.201490278

Radford, L. (2018). *The Emergence of Symbolic Algebraic Thinking in Primary School* (pp. 3–25). https://doi.org/10.1007/978-3-319-68351-5_1

Rea, S. D., Wang, L., Muenks, K., & Yan, V. X. (2022). Students Can (Mostly) Recognize Effective Learning, So Why Do They Not Do It? *Journal of Intelligence*, *10*(4), 127. https://doi.org/10.3390/jintelligence10040127

Reid O'Connor, B. (2024). Methodologies to reveal young Australian Indigenous students' mathematical proficiency. *Mathematics Education Research Journal*, *36*(2), 311–338. https://doi.org/10.1007/s13394-023-00447-z

Rexigel, E., Kuhn, J., Becker, S., & Malone, S. (2024). The More the Better? A Systematic Review and Meta-Analysis of the Benefits of More than Two External Representations in STEM Education. *Educational Psychology Review*, *36*(4), 124. https://doi.org/10.1007/s10648-024-09958-y

Rojo, M., King, S., Gersib, J., & Bryant, D. P. (2023). Rational Number Interventions for Students With Mathematics Difficulties: A Meta-Analysis. *Remedial and Special Education*, *44*(3), 225–238. https://doi.org/10.1177/07419325221105520

Rudaizky, D., Mărcuş, O., Visu-Petra, L., & MacLeod, C. (2023). Anxiety-linked impairment in the ability to recode stimuli. *Emotion*, *23*(3), 814–824. https://doi.org/10.1037/emo0001093

Sandelowski, M., Barroso, J., & Voils, C. I. (2020). Using qualitative metasummary to synthesize qualitative and quantitative descriptive findings. *Research in Nursing & Health*, *30*(1), 99–111. https://doi.org/10.1002/nur.20176

Scheibling-Sève, C., Pasquinelli, E., & Sander, E. (2020). Assessing conceptual knowledge through solving arithmetic word problems. *Educational Studies in Mathematics*, *103*(3), 293–311. https://doi.org/10.1007/s10649-020-09938-3

Spiers, J., Morse, J. M., Olson, K., Mayan, M., & Barrett, M. (2018). Reflection/Commentary on a Past Article: "Verification Strategies for Establishing Reliability and Validity in Qualitative Research." *International Journal of Qualitative Methods*, *17*(1). https://doi.org/10.1177/1609406918788237

Star, J. R., Tuomela, D., Joglar-Prieto, N., Hästö, P., Palkki, R., Abánades, M. Á., Pejlare, J., Jiang, R. H., Li, L., & Liu, R.-D. (2022). Exploring students' procedural flexibility in three countries. *International Journal of STEM Education*, *9*(1), 4. https://doi.org/10.1186/s40594-021-00322-y

Sterner, H. (2024). Using Graphical Representations to Develop Students' Correspondence Relationships and Covariational Thinking in Pattern Generalizations in Primary School. *International Journal of Science and Mathematics Education*. https://doi.org/10.1007/s10763-024-10520-z

Sun, S., Sun, D., & Xu, T. (2023a). The Developmental Progression of Early Algebraic Thinking of Elementary School Students. *Journal of Intelligence*, *11*(12), 222. https://doi.org/10.3390/jintelligence11120222

Sun, S., Sun, D., & Xu, T. (2023b). The Developmental Progression of Early Algebraic Thinking of Elementary School Students. *Journal of Intelligence*, *11*(12), 222. https://doi.org/10.3390/jintelligence11120222

Teare, M. D. (2016). Transparent reporting of research results in eLife. *ELife*, *5*. https://doi.org/10.7554/eLife.21070

Thomas, J., & Harden, A. (2008). Methods for the thematic synthesis of qualitative research in systematic reviews. *BMC Medical Research Methodology*, *8*(1), 45. https://doi.org/10.1186/1471-2288-8-45

Thompson, N. (2023). Vygotskian scaffolding techniques as motivational pedagogy for gifted mathematicians in further education: a diary-interview study. *Journal of Further and Higher Education*, *47*(4), 492–512. https://doi.org/10.1080/0309877X.2022.2142103

Thurtle, V., Leather, A. J., Wurie, H., Foday, E., Samai, M., & Parmar, D. (2021). Strengthening ethics committees for health-related research in sub-Saharan Africa: a scoping review protocol. *BMJ Open*, *11*(8), e046546. https://doi.org/10.1136/bmjopen-2020-046546

Van Wee, B., & Banister, D. (2024). Literature review papers: the search and selection process. *Journal of Decision Systems*, *33*(4), 559–565. https://doi.org/10.1080/12460125.2023.2197703

Wang, Y., & Lewis, S. E. (2020). Analytical chemistry students' explanatory statements in the context of their corresponding lecture. *Chemistry Education Research and Practice*, *21*(4), 1183–1198. https://doi.org/10.1039/D0RP00063A

Wilkie, K. J., & Hopkins, S. (2024). Primary students' relational thinking and computation strategies with concrete-to-symbolic representations of subtraction as difference. *The Journal of Mathematical Behavior*, *73*, 101121. https://doi.org/10.1016/j.jmathb.2023.101121

Winker, M. A., Bloom, T., Onie, S., & Tumwine, J. (2023). Equity, transparency, and accountability: open science for the 21st century. *The Lancet*, *402*(10409), 1206–1209. https://doi.org/10.1016/S0140-6736(23)01575-1

Yıldırım, D., Kırşan, M., Kıray, S., Korhan, E. A., Üstün, Ç., & Aykar, F. Ş. (2019). Nurse Academicians' Attitudes Related to Academic Ethical Values and Related Factors. *Journal of Academic Ethics*, *17*(4), 363–373. https://doi.org/10.1007/s10805-019-09339-z

Zwanch, K. (2022a). Using number sequences to account for differences in generalizations. *School Science and Mathematics*, *122*(2), 86–99. https://doi.org/10.1111/ssm.12516

Zwanch, K. (2022b). Using number sequences to account for differences in generalizations. *School Science and Mathematics*, *122*(2), 86–99. https://doi.org/10.1111/ssm.12516