Koordinat **Jurnal Pembelajaran Matematika dan Sains**
Volume 7, Number 1, January-June 2026, Pages 1-18

https://doi.org/10.24239/koordinat.v7i1.205 **e-ISSN : 2745-4215**

GEOGEBRA CLASSROOM FOR OVERCOMING EPISTEMOLOGICAL OBSTACLES AND ENHANCING MATHEMATICAL LITERACY: A BRUNER PERSPECTIVE THEORY

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ARTICLE INFO

Article History
Received: 14 Feb 2026
Revised: 03 Mar 2026
Accepted: 10 Mar 2026
Published: 19 Apr 2026

Keywords:
Bruner's Representations Theory
Epistemological Obstacles
Exponents
GeoGebra Classroom
Mathematical Literacy

ABSTRACT

This study investigates how GeoGebra Classroom supports students in overcoming epistemological obstacles and enhancing mathematical literacy in learning exponents through Bruner's representational approach. Difficulties in understanding exponent concepts often arise from students' limited ability to connect concrete experiences, visual representations, and symbolic reasoning. This qualitative phenomenological study involved 30 eighth-grade students from a junior high school in Gowa Regency, South Sulawesi, Indonesia. Twelve students were purposively selected to participate in GeoGebra Classroom learning sessions, and three students representing high, medium, and low mathematical abilities were interviewed in depth. Data were collected through participatory observation, semi-structured interviews, and documentation, and analyzed using thematic analysis. The findings show that GeoGebra Classroom facilitates learning through Bruner's enactive, iconic, and symbolic stages. Concrete activities and visual exploration helped students recognize exponential patterns and understand exponent properties. Students with high mathematical ability demonstrated strong skills in formulating, employing, and interpreting mathematical ideas. Students with moderate ability applied procedures correctly but relied on visual feedback from GeoGebra to confirm their reasoning, while students with lower ability experienced difficulties connecting contextual problems with symbolic operationsy. These findings suggest that GeoGebra Classroom provides an effective exploratory environment that supports conceptual understanding and the development of students' mathematical literacy.

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How to cite:
Raisatunnisa, Suryadi, D., Fatimah, S., Priatna, N., Chen, J., & Wicaksono, A. (2026). Geogebra Classroom For Overcoming Epistemological Obstacles And Enhancing Mathematical Literacy: A Bruner Perspective Theory. *Koordinat Jurnal MIPA, 7*(1), 1-18. https://doi.org/10.24239/koordinat.v7i1.205.

INTRODUCTION

The rapid advancement of science and technology in the 21st century has significantly impacted various aspects of life, including education. Innovations in educational technology have created vast opportunities to enhance the quality of learning, particularly in mathematics education, which demands the comprehension of abstract concepts and high-level thinking skills (Bondarenko, 2022). Therefore, the integration of digital technology into mathematics learning has become an important strategy to improve the quality of instruction and support meaningful learning experiences.

Digital technology enables the development of more interactive, collaborative, and contextual learning through various applications and digital platforms (Novaković, 2021). One such platform increasingly utilized in mathematics learning is GeoGebra Classroom. As a technology-based application, GeoGebra Classroom provides mathematical visualizations and simulations that help students grasp complex concepts (Ziatdinov & Valles, 2022). Through dynamic visualization and interactive tasks, GeoGebra enables students to manipulate mathematical objects, observe emerging patterns, and verify mathematical relationships directly. In the context of learning exponents, GeoGebra Classroom serves as a bridge between students' conceptual understanding and the real world, particularly in fostering deep and effective mathematical literacy (Rahmatika, 2023; Sofwatun et al., 2025; Zöchbauer et al., 2021)

The low level of mathematical literacy among Indonesian students remains a major unresolved challenge. According to the 2022 Programme for International Student Assessment (PISA) report by the (OECD, 2023), the average mathematical literacy score of Indonesian students was only 366, far below the global average of 472. Moreover, over 80% of Indonesian students were categorized as low performers in mathematical literacy (Ulul Azmi et al., 2023). A similar situation is also observed at the regional level. In South Sulawesi, particularly in Gowa Regency where this study was conducted, preliminary observations and discussions with mathematics teachers indicate that many junior secondary students still experience difficulties in understanding abstract mathematical concepts and applying them in contextual situations. Students tend to rely on procedural calculations rather than conceptual reasoning when solving problems related to topics such as exponents and algebraic expressions.

This reflects a significant gap between national educational expectations and on the ground realities. This issue is exacerbated by teacher centered instruction and textbook oriented learning (Levitt & Grubaugh, 2023) which often neglect students' critical thinking processes and make mathematics learning monotonous and disconnected from real-life contexts. As a result, students struggle not only with solving mathematical problems but also with recognizing the relevance of mathematics in daily life a key component of mathematical literacy.

In Indonesia's mathematics education, the fundamental issue students face is not merely an inability to solve problems, but epistemological in nature: difficulties in understanding and constructing mathematical knowledge (Herman et al., 2022). This issue was also observed in the research site in Gowa Regency, South Sulawesi. Preliminary classroom observations and analysis of students' written work revealed that many eighth-grade students experienced difficulties in interpreting exponent concepts conceptually. For example, some students interpreted 2^3 as $2 \times 3 = 6$ rather than understanding it as repeated multiplication $2 \times 2 \times 2 = 8$. Other students assumed that $5^0 = 0$ because they associated the exponent zero with the number zero itself. These responses indicate that students often rely on intuitive reasoning rather than conceptual understanding when dealing with exponent rules. These epistemological obstacles include misunderstandings of basic concepts, misuse of procedures, and misinterpretation of mathematical representations. According to (Brousseau, 2005) such obstacles emerge from a disconnect between the scientific knowledge being taught and the spontaneous knowledge students possess. Therefore, instructional approaches that combine conceptual exploration with interactive representations are required to help students reconstruct their mathematical understanding. Digital learning environments such as GeoGebra Classroom offer opportunities to support this process by enabling students to explore mathematical ideas through dynamic visualizations and guided experimentation.

Mathematical literacy refers to an individual's ability to formulate, apply, and interpret mathematics in various contexts, including real life. It involves not only mastering concepts and procedures, but also reasoning, problem-solving, and communicating mathematical ideas (Oktradiksa et al., 2023). According to the National Council of Teachers of Mathematics (NCTM), it comprises five key competencies: problem-solving, reasoning, communication, connections, and mathematical representation. In practice, students with high mathematical literacy can identify real-world problems, translate them into mathematical forms, solve them using appropriate procedures, and meaningfully interpret the results (Rizaldi et al., 2023). However, these competencies remain underdeveloped due to teaching approaches that fail to stimulate such skills effectively(Vasile, 2022).

Various instructional approaches have been developed to improve mathematical literacy, including contextual, problem-based, and technology assisted learning. One effective approach is the use of digital media such as GeoGebra in mathematics instruction (Rahmawati et al., 2021). GeoGebra is an interactive software that integrates graphics, algebra, and calculus within a single platform. In learning about exponents, GeoGebra allows students to visualize concepts, explore patterns, and construct knowledge independently (Dahal et al., 2022; Díaz-Berrios & Martínez-Planell, 2022; Suci & Reflina, 2023). Research by Uwurukundo et al, (2020) has shown that using GeoGebra significantly improves students' mathematical literacy. These studies highlight the role of dynamic digital tools in supporting visualization, exploration, and conceptual learning in mathematics.

However, the majority of previous studies primarily focus on the effectiveness of GeoGebra in improving learning outcomes, students' motivation, or conceptual understanding, rather than examining how the technology can be systematically integrated into a learning design that addresses students' epistemological obstacles during the knowledge construction process.

Furthermore, although several studies have discussed the integration of GeoGebra in mathematics learning, there remains a conceptual gap concerning how technology-based learning environments can facilitate students' cognitive development through representational stages while simultaneously addressing epistemological obstacles that hinder mathematical literacy development. Most existing research tends to treat GeoGebra merely as a visualization tool, without explicitly connecting its use with a theoretical framework of knowledge representation such as Bruner’s enactive, iconic, and symbolic stages (Batiibwe, 2024; Medina-Hidalgo et al., 2025; Sari et al., 2024) Consequently, there is limited understanding of how students construct mathematical meaning through progressive representational experiences when interacting with digital learning environments.

Therefore, this study offers a novel contribution by integrating the use of GeoGebra Classroom with Bruner’s representational theory to explore how technology-supported learning can help students overcome epistemological obstacles while simultaneously fostering mathematical literacy. Unlike previous studies that primarily evaluate learning outcomes, this research focuses on analyzing students’ learning experiences and cognitive processes during the transition from enactive to iconic and symbolic representations in the topic of exponents

Based on this background, this article aims to analyze and describe students’ experiences using GeoGebra Classroom to overcome epistemological obstacles and enhance mathematical literacy in the topic of exponents, viewed through the lens of Bruner’s representational theory. The main focus is to examine how representation-based learning stages can foster deeper conceptual understanding and how students build mathematical literacy through digital exploration. It is hoped that this article will contribute both theoretically and practically to the development of technology-integrated mathematics learning models that are cognitively responsive and capable of addressing mathematical literacy challenges in a comprehensive way.

METHOD

This study employed a qualitative approach with a phenomenological design to explore students' learning experiences using GeoGebra. The phenomenological design is appropriate because it emphasizes the exploration of individuals’ lived experiences related to a particular phenomenon (Dodgson, 2023). In this context, the phenomenon refers to students’ experiences in learning exponents through GeoGebra Classroom within the framework of Bruner’s representational stages (enactive, iconic, and symbolic). Through this approach, the study seeks to capture how students perceive, interpret, and construct mathematical understanding during technology-supported learning activities. Classroom as a mathematics learning medium, particularly in exponent material. The phenomenological approach was chosen to understand the subjective meanings constructed by students when interacting with digital learning media, especially in overcoming epistemological barriers and developing mathematical literacy. Students' learning experiences were understood not only as the result of the learning process, but also as the construction of meaning formed through interactions between students, mathematical concepts, and the GeoGebra digital environment.

The research questions that guided this study include: (1) how are students' experiences in using GeoGebra Classroom to understand exponents; (2) how do Bruner's enactive-iconic-symbolic representations appear in the students' learning process; and (3) how does the use of this digital media contribute to students' efforts in overcoming epistemological barriers and developing mathematical literacy.

This study was conducted at a public junior high school in Gowa Regency, South Sulawesi, Indonesia. The total population consisted of 30 eighth-grade students. Using purposive sampling, 12 students were purposively selected to participate in three learning sessions using GeoGebra Classroom. The selection was based on the following criteria: (1) prior experience in using GeoGebra, (2) willingness to actively participate in learning activities, and (3) recommendations from mathematics teachers regarding students’ engagement in classroom learning. From these participants, three students were selected for in-depth interviews to represent low, medium, and high levels of mathematical ability. This stratified selection aimed to ensure diverse perspectives and to generate rich descriptions of students’ meaning-making processes in digital learning contexts.

Four types of instruments were used in this study. First, GeoGebra Classroom activity sheets were developed based on Bruner's stages of repreentation (enactive, iconic, and symbolic) and functioned as learning stimuli to elicit students' learning experiences. Second, a mathematical literacy test consisting of contextual problems related to exponents and roots was administered to examine students' abilities to formulate, employ, and interpret mathematics based on the PISA framework. Third,semi-structured interview guidelines were designed to explore students' conceptual understanding, learning experiences, and epistemological barriers. Fourth, an observation sheet was employed to document students' interactions, engagement, and responses during GeoGebra-based learning activities. he validity of the instruments was ensured through expert judgment involving three mathematics education experts, and the results indicated that the instruments were considered valid for use in this study, supported by data triangulation, and member checking.

The research procedures were conducted in three main stages. The first stage was orientation, in which the researcher provided a brief introduction to the use of GeoGebra Classroom and provided students with access accounts. The second stage was implementation, which lasted approximately 80 minutes and involved guided exploration, visual representation through iconic models, and the formulation of symbolic generalizations. During this stage, students actively interacted with dynamic digital content and completed structured learning tasks. The third stage was reflection, which involved in-depth interviews and reflective discussions to explore students' learning experiences, conceptual understanding, and epistemological challenges encountered during the learning process.The research procedures were conducted in three main stages. The first stage was orientation, in which the researcher provided a brief introduction to the use of GeoGebra Classroom and provided students with access accounts. The second stage was implementation, which lasted approximately 80 minutes and involved guided exploration, visual representation through iconic models, and the formulation of symbolic generalizations. During this stage, students actively interacted with dynamic digital content and completed structured learning tasks. The third stage was reflection, which involved in-depth interviews and reflective discussions to explore students' learning experiences, conceptual understanding, and epistemological challenges encountered during the learning process.

Data were collected through participatory observation, semi-structured interviews, and documentation of student activities on the GeoGebra Classroom platform. The research instruments consisted of an observation sheet, a semi-structured interview guide, and a documentation protocol used to record students' learning activities during the implementation of GeoGebra Classroom. Prior to data collection, the instruments were validated through expert judgment by two experts in mathematics education to ensure the content validity, relevance, and clarity of each instrument. Observations focused on students' interactions with digital content, responses to automated feedback, and classroom discussions. Interviews were conducted after learning sessions, lasted approximately 30 to 45 minutes, were audio-recorded with participants' consent, and transcribed verbatim. Documentation included screenshots, learning logs, and students' digital work records

Data were analyzed using Creswell's (2014) phenomenological framework. Observation notes, interview transcripts, and documentation were organized and reviewed, followed by coding of significant statements. These codes were clustered into themes. Textural and structural descriptions were then developed, and the essence of students' learning experiences was synthesized to interpret their meaning-making processes.

RESULT AND DISCUSSION

The Use of Bruner's Theory in Utilizing GeoGebra Classroom for Teaching Exponents.

Result

At the enactive stage, students accessed GeoGebra Classroom by entering an instructional code provided by the teacher and worked on digital worksheets that could be monitored in real time. The initial activity included stated learning achievements, in which students were expected to read, write, and compare exponential and root numbers, as well as to understand the basic concepts and properties of exponents. Students were then asked to perform a paper-folding activity using sheets previously distributed by the teacher.

They folded the paper repeatedly to divide it into equal parts and observed the resulting sections. A response box was provided in the worksheet to replace conventional answer sheets, allowing students to record the number of sections formed and identify the corresponding multiplication patterns. After constructing an initial understanding through paper folding, students were given a contextual problem related to bacterial growth, in which bacteria were described as dividing into two every hour. Using digital tools such as pens, rulers, and erasers available on the platform, students illustrated bacterial growth up to the third hour in the GeoGebra activity sheet (Figure 1)

Figure 1. Worksheet in GeoGebra Classroom

The activities carried out by students above are classified as the enactive stage in Bruner's learning theory, in which students initially learn the concept of exponents using concrete objects (for example, folding a sheet of paper into several rectangular parts of equal size, then counting how many folds produce how many rectangles, and writing down the multiplication pattern to derive the concept of exponentiation). In the second activity, using tools provided in the digital worksheet, students illustrated bacterial growth over several hours

At the iconic stage, students constructed new knowledge by using information provided in the GeoGebra Classroom activities. The learning tasks were designed to represent exponent concepts through visual and interactive elements without explicitly defining the rules at the beginning. Students accessed the third activity using an instructional code, and their progress was monitored by the teacher in real time. The learning objective at this stage was to develop students' understanding of exponent properties. The digital worksheets presented various rules, including positive and negative integer exponents, zero exponents, exponentiation of powers, multiplication and division of exponents, and exponentiation of products and quotients.

Students were instructed to input arbitrary values for p and n and observe how changes in input affected the outcomes. Through systematic exploration and observation, students identified patterns in the results and were guided to generalize formulas related to exponent properties. This process supported the development of visual reasoning and helped students connect experimental findings with formal mathematical concepts.

At the symbolic stage, students were required to apply their conceptual understanding to solve problems involving exponent operations. Students accessed the learning tasks through instructional codes, and their progress was continuously monitored by the teacher. The digital activity sheets provided answer boxes that replaced conventional worksheets and contained contextual word problems related to mathematical literacy. These problems required students to apply exponent rules in real-life situations, such as calculating quantities and comparing magnitudes. By utilizing the properties of exponents developed in previous stages, students were able to perform symbolic manipulations, interpret results, and formulate logical conclusions. This stage emphasized abstract reasoning and the integration of symbolic representation with contextual understanding.

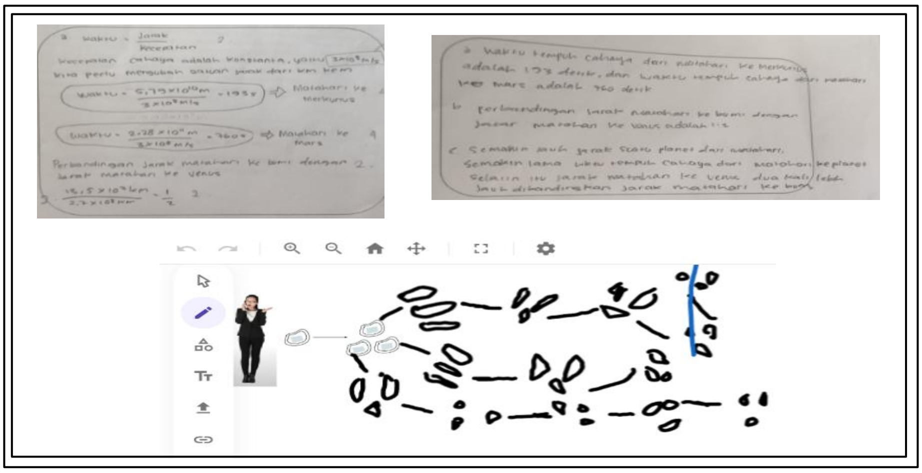

Figure 1. Work Result of Subject 1

To support the data presented above, the following is the interview transcript with Subject S1

a) Enactive stage concrete experience through physical and visual activities

Teacher :Let's start from when you folded the paper and drew the bacterial growth in GeoGebra Classroom. What did you learn from those activities?

S1 : I noticed that every time I folded the paper, the number of sections doubled. The same thing happened when I drew the bacteria from one to two, then to four. At first, I thought it was just increasing, but then I realized it was like powers of two, powers of three.

Teacher : Were you immediately able to connect that to exponentiation?

S1 : Not right away. But since I could see the physical shape and the drawings in GeoGebra, I started to understand that exponents represent multiplication or duplication.

b) Iconic stage connecting visualizations and semi concrete representations

Teacher :When you started entering numbers in GeoGebra to explore the properties of exponents, how did you feel?

S1 :I tried entering random numbers like 2^4 or 2^{-1} and looked at the results. When the result suddenly became very small, I got curious. I thought the higher the exponent, the bigger the number, but it turns out it can get smaller if the exponent is negative.

Teacher: How did you conclude the properties?

S1 : I looked at the results in GeoGebra. For example, when 2^0 equals 1, I tried with other numbers and noticed a pattern. Then I wrote the formulas myself.

c) Symbolic stage solving mathematical literacy problems

Teacher: You worked on a problem about the travel time of light using exponents. Can you explain how you solved it?

S1 :I used the formula time = distance ÷ speed. Then I input the exponents and subtracted them: $10^8 \div 10^5$ became

10^3. I also divided the leading numbers.

Teacher: When you compared the distance between Earth and Venus, you wrote the ratio as 1:2. Were you confident with that result?

S1 :Yes, because GeoGebra helped show it too. I saw the exponents were the same, so I just divided the front numbers. The result was 0.5, which means Venus is twice as far.

Teacher: If you were asked to explain it to a friend, would you prefer to use a graph or numbers?

S1 :If possible, both. I'd show them the graph in GeoGebra and then explain with numbers so it's easier to understand.

The use of GeoGebra Classroom significantly helped the student understand the concept of exponents by overcoming the epistemological obstacles often encountered in the early stages of learning. Through the enactive approach, the student engaged in concrete activities such as folding paper and illustrating bacterial growth, which revealed patterns of duplication. This direct experience helped them recognize the link between real-world phenomena and exponential concepts, although some confusion initially arose when connecting physical processes with symbolic notation. Gradually, however, a deeper conceptual understanding began to emerge.

In the iconic and symbolic stages, GeoGebra provided interactive visualizations that allowed the student to explore exponent properties independently. By directly observing changes in values and outcomes, the student was able to draw conclusions about key rules, such as zero exponents, negative exponents, and operations like exponent multiplication and division. This encouraged inductive reasoning and helped correct prior misconceptions. Ultimately, the student demonstrated enhanced mathematical literacy, shown by their ability to solve contextual problems, interpret results in real-world contexts, and express logical, structured reasoning.

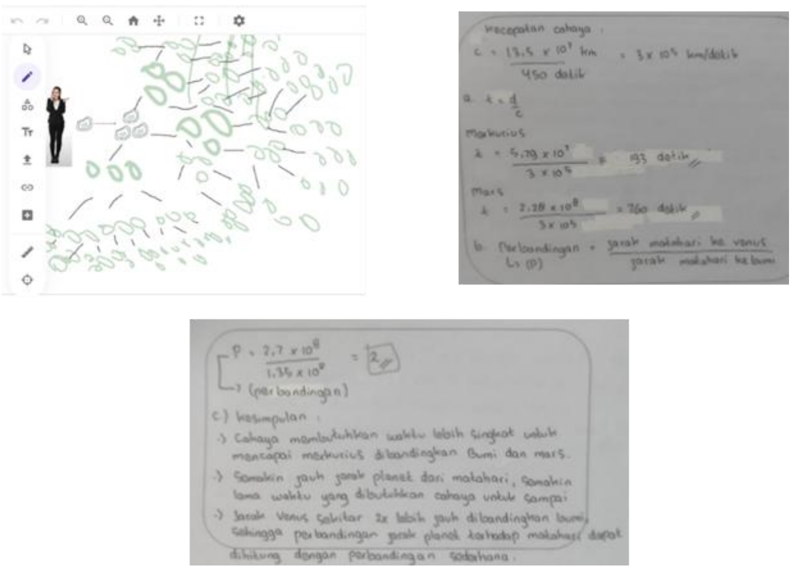

Figure 2. Work Result of Subject 2

To support the data above, the following is the interview transcript with Subject S2

a) Enactive Stage Paper Folding Activity and Bacterial Growth

Teacher: At the beginning of the lesson, you did a paper-folding activity to learn about exponents. Can you explain what you understood from that activity?

Student: I saw that every time I folded the paper, the number of sections increased. Like, it started with one, then two, then four, then eight… so I wrote it like 2 to the power of 1, 2 to the power of 2, and so on.

Teacher: When you wrote that pattern, did you immediately understand that it was exponentiation?

Student: Not at first, I thought it was just doubling. But after seeing the result and matching it with the teacher's explanation in GeoGebra, I understood that it could be written using exponents.

Teacher: Was there any part that made you unsure while folding the paper and writing the pattern?

Student: Yes, I was confused at first why it was called power of one when the result was two. But after calculating and getting the explanation, I started to get it.

b) Iconic stage exploring exponent properties in GeoGebra

Teacher: After that, you went into the GeoGebra activity showing the properties of exponents. What did you do there?

Student: I tried inputting random values for p and n, then observed the results on the screen. I experimented until I understood the pattern.

Teacher: Did you feel confused at any point when seeing the results?

Student: I was confused when I encountered negative exponents because the result turned into a fraction. But GeoGebra helped me understand since I could see the results immediately.

Teacher: Were you able to conclude any formulas from that activity?

Student: Yes. I concluded that if you multiply, you add the exponents. If you divide, you subtract. A zero exponent equals one. I saw all that from how the values changed.

c) Symbolic stage solving contextual problems

Teacher: In the problem where you compared distances between planets and the time light takes to travel, how did you solve it?

Student: I used the formula time equals distance divided by speed. I divided 5.79×10^7 by 3×10^5, and the result was 193 seconds.

Teacher: When you performed the exponent operation, were you confident that your steps were correct?

Student: I double-checked. I remembered that when dividing, you subtract the exponents. GeoGebra also helped me verify the final result.

Teacher: In the comparison part, you wrote the final result as 2. What does that mean to you?

Student: It means the distance to Venus is twice as far as the distance from Earth to the Sun. I concluded that from dividing the leading numbers since the exponents were the same.

Teacher: Do you think it's important to write down conclusions like the one you just made?

Student: Yes, because then we understand what the numbers actually mean. If we don't explain, we only know the result, not the meaning behind it.

Based on observations and the interview, GeoGebra Classroom proved effective in helping students overcome epistemological obstacles in understanding exponents. In the enactive stage, students engaged in concrete activities such as folding paper and observing bacterial growth, which triggered intuitive understanding of exponential concepts. The interactive visualizations helped students realize that the process of doubling could be represented exponentially, enabling them to gradually grasp the meaning of exponential notation.

In the subsequent iconic and symbolic stages, students explored various properties of exponents through manipulating values and observing outputs in GeoGebra. Visual features such as graphs and tables reinforced their understanding of exponent rules and helped address common misconceptions, including those related to zero and negative exponents. Students were able to solve contextual problems and interpret the results logically, indicating improved mathematical thinking skills. GeoGebra served not only as a calculation tool but also as a medium for developing comprehensive mathematical literacy and reasoning.

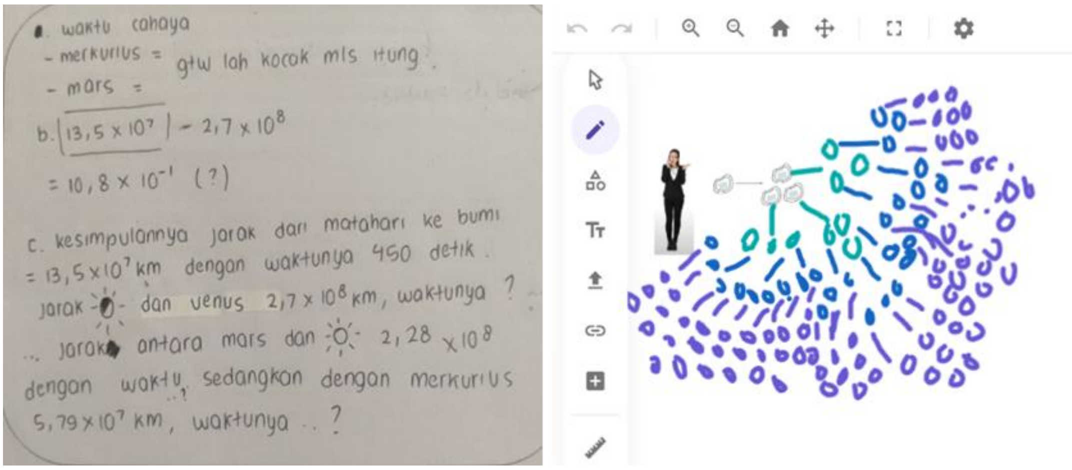

Figure 3. Work Result of Subject 3

To support the data above, the following is the interview transcript with Subject S3

a) Enactive stage paper folding & bacterial growth activity

Teacher: Do you remember when we folded the paper at the beginning of the lesson? What did you learn from that?

Student: Yes, it was for learning about exponents. I saw that each time I folded it, the number of parts increased. But I didn't really know how to count it when it got too many.

Teacher: Were you confused when asked to write the folding pattern as a multiplication?

Student: Yes, I thought you just had to add them, not multiply. Because I saw the number of sections increase one by one.

b) Iconic stage exploring exponent properties with GeoGebra

Teacher: When you entered numbers into GeoGebra to explore exponents, what did you notice?

Student: I saw the result changed right away. But I didn't understand why it got smaller when the exponent was negative.

Teacher: Did you try to make conclusions, like "when you multiply, you add the exponents," or something like that?

Student: I saw there was a pattern, but I got confused when I put in zero or negative numbers. I felt like I kept getting it wrong.

c) Symbolic Stage Solving Mathematical Literacy Problems

Teacher: When you were asked to calculate the travel time of light to Mercury and Mars, you wrote "I don't know, this is too much, I don't feel like calculating." Can you tell me why you wrote that?

Student: Because I was really confused. I wasn't sure how to divide it, what to do with the exponents, and I didn't get the units either. So I just left it.

Teacher: When asked to find the distance ratio, you wrote 10.8×10^{-1}. Were you confident with that answer?

Student: Not really. I just followed what I saw in GeoGebra, but I think the number was off. I was confused because I didn't understand how to divide the exponents.

Teacher: If you were asked to explain your answer to a friend, could you do it?

Student: I'm not sure. I understand it better if someone shows me again or explains it using a graph.

Observation and interview results show that using GeoGebra Classroom offers a rich learning experience for understanding exponent concepts, even though not all students were able to fully overcome epistemological obstacles. In the enactive stage, students engaged in hands-on activities such as folding paper and simulating bacterial growth to develop initial intuitions about exponents as repeated multiplication. While these activities were effective in supporting early understanding, some students still struggled to distinguish between additive and exponential growth indicating that the transition from concrete experience to abstract understanding was not yet fully developed in all learners.

In the iconic stage, GeoGebra's interactive visualizations helped students recognize patterns and properties of exponents, such as zero exponents, negative exponents, and operations involving division. However, some students remained confused by non-intuitive results, especially with negative exponents. This highlights that conceptual understanding still depends heavily on the students' ability to reflect and make sense of their exploratory results. At the symbolic stage, students were challenged to apply their conceptual understanding to mathematical literacy problems such as calculating light travel time or comparing interplanetary distances. While some students were able to interpret and draw conclusions successfully, others showed resistance or avoidance when facing symbolic tasks, suggesting that epistemological obstacles were not yet fully resolved. Overall, the effectiveness of GeoGebra Classroom depends largely on students' ability to meaningfully connect concrete, visual, and symbolic representations.

Discussion

This discussion section addresses the research questions that guided this study: (1) how students experience learning exponents using GeoGebra Classroom; (2) how Bruner's enactive, iconic, symbolic representations emerge throughout their learning process; and (3) how the use of this digital media contributes to students' efforts in overcoming epistemological barriers and developing mathematical literacy. Overall, the findings show that students progressed through the three representational stages with varying degrees of conceptual clarity, and that GeoGebra Classroom played a significant role in supporting understanding, correcting misconceptions, and fostering reflective thinking

1. Enactive stage: Understanding exponents through concrete and interactive activities

In the enactive stage, students learned through direct experience via concrete activities in GeoGebra Classroom designed to stimulate action-based representation. Activities such as folding paper and illustrating bacterial growth served to embody the concept of multiplication and exponential growth. This aligns with Bruner's theory that effective learning begins with concrete experiences before moving to abstract representation. Some students faced epistemological obstacles in interpreting folding patterns as additive instead of exponential. However, through interactive visuals, students were able to revise these perceptions and form stronger conceptual understanding. GeoGebra enabled autonomous exploration and reinforced the connection between physical experience and mathematical ideas as Zetriuslita et al, (2020) pointed out, using GeoGebra in action-based learning can reduce initial anxiety around abstract material. These findings are also consistent with Sheng et al (2023) who emphasized the importance of concrete stages in building experiential mathematical literacy.

2. Iconic stage: Constructing visual representations to understand exponent properties

In the iconic stage, students transitioned to visual representations to process information gathered from direct experience. According to Bruner's theory of representation, the iconic stage enables learners to construct meaning through images and visual models that bridge concrete experiences and abstract reasoning (Bruner, 1966). GeoGebra Classroom provided exploratory tasks that allowed students to test different values and immediately view the outcomes of exponential operations. Graphs, tables, and digital models helped students observe consistent patterns such as the results of raising numbers to zero or negative powers. Previous research has shown that visual representations can support conceptual understanding by enabling students to identify patterns and relationships within mathematical structure (Duval, 2006; Just & Cribbs, 2020)

Some students successfully generalized from these observations, showing that visual representation supports deep understanding and helps correct misconceptions. However, others still encountered epistemological obstacles, particularly when the results did not align with their intuitive expectations. This underscores the need for pedagogical scaffolding to assist students in transforming visual observations into meaningful symbolic formulas. The study demonstrates that success at this stage depends on active engagement and reflection during digital experimentation. Supporting this Nahum-Shani et al (2022) states that GeoGebra is a powerful tool for bridging the transition from visual intuition to symbolic abstraction, especially when students are provided guided freedom in their exploration.

3. Symbolic stage: Applying concepts in mathematical literacy contexts

The symbolic stage marks the highest level of cognitive development in Bruner's hierarchy, where students operate with mathematical symbols to solve real-world problems. In this study, students were challenged with contextual tasks such as calculating the light travel time between planets and comparing astronomical distances using exponents. GeoGebra helped clarify principles of exponential operations like division, and supported logical interpretation of results. Some students successfully connected symbolic operations with contextual meaning, reflecting strong mathematical literacy skills. However, a number of students rejected the symbolic tasks due to lack of confidence, showing that epistemological obstacles were still present for some. This aligns with findings by Malhi et al (2021) who argue that symbolic understanding depends on prior ability to connect with visual and concrete representations. Similarly, Botha et al (2019) stress the importance of integrative approaches that blend visual, symbolic, and narrative models to reinforce mathematical literacy. In this context, GeoGebra serves not merely as a calculator, but as a cognitive bridge for comprehensive mathematical reasoning.

4. Theoretical framing: Learning with representations and overcoming epistemological obstacles

This study explored how students construct mathematical meaning through the stages of Bruner’s representation enactive, iconic, and symbolic supported by the digital learning environment of GeoGebra Classroom. By embedding students' learning trajectories within this theoretical structure, the research highlights how mathematical understanding can emerge from concrete engagement, be refined through visualization, and solidified through abstract symbolic reasoning. Previous studies have also emphasized that dynamic mathematics software such as GeoGebra can facilitate conceptual understanding by enabling students to explore mathematical relationships through interactive representations (Azis & Rohaeti, 2025; Medina-Hidalgo et al., 2025). These findings also reinforce the centrality of addressing epistemological obstacles deep-seated conceptual misunderstandings as part of the cognitive transformation process in mathematics education.

5. Connections with prior literature

The results resonate with previous studies that emphasize the power of digital environments to bridge gaps in mathematical understanding. For instance, (Zetriuslita et al., 2020) found that action-based activities using GeoGebra reduce students’ anxiety toward abstract content. Sheng et al (2023) emphasized that early concrete experiences are essential for fostering mathematical literacy. Moreover, Nahum-Shani et al (2022) demonstrated how GeoGebra can scaffold transitions from visual intuitions to formal symbolic thinking. The present study extends these insights by applying them specifically to the topic of exponents, which is often conceptually challenging for junior secondary students.

The findings of this study can also be interpreted through broader theoretical perspectives on mathematical learning. From a constructivist viewpoint, students build understanding through active engagement with tasks and representations. This was evident in the learning trajectories observed in this study, where students gradually developed their understanding of exponent concepts through exploration and interaction within the GeoGebra Classroom environment. These results support (Bruner, 1966) representational theory, which suggests that conceptual understanding develops through transitions from enactive experience to iconic visualization and finally to symbolic abstraction. The persistence of misconceptions in some students, particularly when interpreting zero and negative exponents, can also be explained by Brousseau's concept of epistemological obstacles, where prior intuitive knowledge conflicts with formal mathematical structures. Similar patterns have been reported in previous studies, which highlight the role of dynamic mathematics software in facilitating conceptual learning and mathematical literacy (Nahum-Shani et al., 2022; Sheng et al., 2023; Zetriuslita et al., 2020). Thus, the present study not only confirms these theoretical perspectives but also extends prior research by demonstrating how GeoGebra Classroom can function as a representational bridge that helps students overcome epistemological barriers while developing mathematical literacy in learning exponents

6. Unexpected results: Persistent obstacles in the symbolic stage

While most students showed growth in understanding through the enactive and iconic stages, some exhibited resistance or confusion when engaging with symbolic representations. Despite exposure to exploratory visual tasks, these students struggled to internalize rules such as the meaning of zero and negative exponents. Instead of applying symbolic logic, they defaulted to intuitive but incorrect strategies. This finding suggests that even well-structured digital interventions require intentional scaffolding when students transition from visual understanding to symbolic reasoning. Such difficulties are consistent with the findings of Malhi et al. (2021), who argue that mastery of symbolic mathematics depends on students' prior ability to connect symbolic rules with concrete and visual representations. Similarly, Sfard (1991) emphasizes that conceptual development in mathematics requires a gradual shift from operational understanding to structural abstraction, which cannot occur without sufficient representational support. Therefore, the persistence of difficulties at the symbolic stage in this study highlights the importance of carefully designed instructional scaffolding to help students consolidate connections between visual exploration and formal symbolic reasoning.

7. Mathematical Literacy Analysis Based on the PISA Framework Supported by GeoGebra

To further analyze students' learning outcomes, their responses were examined using the mathematical literacy processes proposed in the PISA framework, namely formulating, employing, and interpreting mathematics. These processes were identified through students' problem-solving strategies during contextual tasks and their interactions with the GeoGebra Classroom environment. The dynamic features of GeoGebra allow students to explore mathematical relationships visually and verify the results of symbolic operations, which can support the development of mathematical literacy (Nahum-Shani et al., 2022; Zetriuslita et al., 2020)

Subject S1 (High Mathematical Ability) demonstrated strong performance across the three mathematical literacy processes with the support of GeoGebra Classroom. In the formulating stage, the student was able to translate contextual problems into mathematical expressions involving exponents by identifying relevant quantities and relationships. The visual feedback from GeoGebra helped the student confirm the patterns of exponential growth observed during exploration activities. In the employing stage, S1 correctly applied exponent rules, such as subtracting exponents during division operations and manipulating coefficients accurately. The interactive environment enabled the student to verify the results of calculations and refine their reasoning. In the interpreting stage, the student successfully explained the meaning of the results in real-world contexts, such as interpreting ratios of astronomical distances. This finding is consistent with research showing that dynamic mathematics software can enhance conceptual understanding and mathematical reasoning by connecting symbolic manipulation with visual exploration (Dockendorff, 2019; Zengin, 2019)

Subject S2 demonstrated developing mathematical literacy supported by GeoGebra Classroom. In the formulating stage, the student was able to identify relevant mathematical relationships from contextual problems presented in the digital worksheets. GeoGebra's interactive environment allowed the student to experiment with different numerical inputs and observe the resulting exponential patterns. During the employing stage, the student applied exponent rules correctly in many cases, although the student often relied on GeoGebra to confirm the correctness of the procedures. In the interpreting stage, the student was able to explain the meaning of the results, although the explanations tended to focus on procedural steps rather than deeper conceptual reasoning. These findings align with previous studies suggesting that digital visualization tools can support students' understanding of mathematical concepts by enabling exploratory learning and immediate feedback (Sheng et al., 2023)

Subject S3 experienced significant challenges across the three mathematical literacy processes despite the support provided by GeoGebra Classroom. In the formulating stage, the student struggled to translate contextual situations into mathematical representations involving exponents. Although GeoGebra displayed visual patterns and numerical results, the student found it difficult to connect these representations with formal mathematical expressions. In the employing stage, the student showed uncertainty when performing exponent operations, particularly when dividing exponential expressions and interpreting negative exponents. In the interpreting stage, the student was unable to confidently explain the meaning of the results within the given context. These findings indicate that while digital tools can support conceptual learning, students with lower prior knowledge may still require additional scaffolding to connect visual representations with symbolic reasoning. Similar findings were reported by Malhi et al., (2021), who noted that difficulties in symbolic reasoning often occur when students fail to link concrete and visual representations with formal mathematical structures.

8. ClassroomImplications for practice and theory

Practically, this study shows that designing lessons that incorporate GeoGebra in tandem with Bruner's representational stages can improve conceptual learning outcomes and reduce epistemological barriers. Educators should sequence digital tasks to allow for progressive abstraction, starting from hands-on exploration before advancing to symbolic application. Theoretically, these results support a view of digital mathematics education not simply as technology use, but as a medium for epistemic engagement where learners actively reconstruct knowledge through structured representational shifts. This reinforces the pedagogical value of representational theory and supports calls for integrative designs in mathematics instruction.

The findings of this study offer several important implications. Theoretically, the results confirm the relevance of Bruner's representation theory within technology-enhanced learning, showing that digital platforms can strengthen the transition from concrete experience to symbolic reasoning. Practically, the study suggests that GeoGebra Classroom can serve as an effective pedagogical tool to scaffold students' understanding of exponents, especially when addressing epistemological barriers such as misinterpreting multiplicative patterns. For mathematics literacy development, the structured progression across the enactive–iconic–symbolic stages helps students connect real-world representations, visual reasoning, and symbolic generalization.

This study has several limitations that should be acknowledged. First, the learning activities were implemented within a limited time frame, which may have constrained students' opportunities for deeper exploration. Second, the sample size was relatively small, limiting the generalizability of the findings to broader populations.

CONCLUSION

Based on the results of this study, it can be concluded that the use of GeoGebra Classroom effectively supports the mathematics learning process particularly on the topic of exponents through a gradual approach aligned with Jerome Bruner's theory of representation. Through the enactive, iconic, and symbolic stages, students are facilitated to construct conceptual understanding progressively: beginning with concrete experiences, continuing with visualization, and finally operationalizing the concepts in symbolic and applied forms. This approach proves to reduce epistemological obstacles that commonly arise due to misconceptions or limitations in understanding abstract mathematical notation.

Furthermore, GeoGebra Classroom offers an exploratory and interactive environment that supports the development of students' mathematical literacy, including problem formulation, idea representation, and result interpretation in real-world contexts. These abilities correspond to the mathematical literacy processes in the PISA framework, namely formulating, employing, and interpreting mathematics. The findings indicate that students demonstrate different levels of mathematical literacy development depending on their level of ability. Students with high mathematical ability were able to translate contextual problems into mathematical representations, apply exponent rules accurately, and interpret the results meaningfully within real-world contexts. Students with moderate ability were able to formulate and employ mathematical procedures correctly, although they still relied on visual feedback from GeoGebra Classroom to confirm their reasoning. Meanwhile, students with lower mathematical ability experienced difficulties in connecting contextual problems with symbolic operations and interpreting the results, indicating that epistemological obstacles were still present in the transition toward symbolic reasoning.

Its dynamic features strengthen students' logical and reflective thinking skills in solving contextual math problems. The interactive visualization and immediate feedback provided by GeoGebra Classroom also help students explore patterns of exponent operations and verify their mathematical reasoning, thereby supporting the development of mathematical literacy.However, the effectiveness of GeoGebra also depends on students' cognitive readiness and the quality of pedagogical interactions throughout the learning process. Therefore, integrating GeoGebra Classroom based on Bruner's approach is recommended as an innovative strategy to design more meaningful mathematics instruction that enhances both conceptual understanding and holistic mathematical literacy.

To support further development of this research, future studies may investigate the application of GeoGebra Classroom with Bruner's representational stages in different mathematical topics or across various grade levels to examine its broader effectiveness. Comparative studies involving different digital platforms or instructional models could also provide deeper insights into how technological tools interact with students' cognitive processes and contribute to reducing epistemological barriers. In addition, longitudinal research may offer a more comprehensive understanding of how sustained use of GeoGebra influences the long-term development of students' mathematical literacy.

ACKNOWLEDGMENT

We express our gratitude to LPDP (Lembaga Pengelola Dana Pendidikan) for providing the scholarship funding that supported the execution of this study.

AUTHOR CONTRIBUTIONS

Author One: conceptualization, data curation, formal analysis, investigation, methodology, and visualization. She also took primary responsibility for writing the original draft and contributed to review and editing;
Author Two: supervision (oversight and academic guidance), project administration (coordination and management of research activities), and validation (verification and reproducibility of research findings), as well as writing review.
Author Three: supervision (research oversight), resources (provision of research materials and resources), and validation (verification of results), in addition to writing review.
Author Four: supervision (academic oversight), software (development and implementation of software tools for the study), and validation (verification of findings), as well as writing review & editing;
Author Five: formal analysis, visualization as well as writing review.
Author Six: formal analysis, data curation, and editing.

REFERENCES

Awaji, B. M., Khalil, I., & AL-Zahrani, A. (2025). A Bibliometrics Study of Two Decades of Geogebra Research in Mathematics Education. *Journal of Educational and Social Research*, *15*(1). https://doi.org/10.36941/jesr-2025-0011

Azis, Y. M., & Rohaeti, E. E. (2025). A systematic literature review on implementation of GeoGebra: Benefits and challenges in mathematics education. *Infinity Journal*, *14*(3). https://doi.org/10.22460/infinity.v14i3.p655-672

Batiibwe, M. S. K. (2024). Integration of GeoGebra in Learning Mathematics: Benefits and Challenges. *East African Journal of Education Studies*. https://doi.org/https://doi.org/10.37284/eajes.7.4.2454

Bondarenko, D. (2022). Critical Thinking Technology As A Tool For Innovative Activity In Mathematics Lessons. *Baltic Journal of Legal and Social Sciences*, *2*. https://doi.org/10.30525/2592-8813-2022-2-4

Botha, H., Van Putten, S., & Kundema, I. (2019). Enhancing visual literacy in the mathematics classroom: The case of Dar es Salaam. *Perspectives in Education*, *37*(2). https://doi.org/10.18820/2519593X/pie.v37i2.8

Brousseau, G. (2005). *The Study of the Didactical Conditions of School Learning in Mathematics*. https://doi.org/DOI:10.1007/0-387-24270-8_14

Bruner, J. s. (1966). *Toward a Theory of Instruct_Bruner*. 1–21.

Dahal, N., Pant, B. P., Shrestha, I. M., & Manandhar, N. K. (2022). Use of GeoGebra in Teaching and Learning Geometric Transformation in School Mathematics. *Int. J. Interact. Mob. Technol.*, *16*, 65–78. https://doi.org/DOI:10.3991/ijim.v16i08.29575

Díaz-Berrios, T., & Martínez-Planell, R. (2022). High school student understanding of exponential and logarithmic functions. *Journal of Mathematical Behavior*, *66*. https://doi.org/10.1016/j.jmathb.2022.100953

Dockendorff, M. (2019). How can digital technology enhance mathematics teaching and learning? In *Examining Multiple Intelligences and Digital Technologies for Enhanced Learning Opportunities*. https://doi.org/10.4018/978-1-7998-0249-5.ch011

Dodgson, J. E. (2023). Phenomenology: Researching the Lived Experience. *Journal of Human Lactation*, *39*(3). https://doi.org/10.1177/08903344231176453

Duval, R. (2006). A Cognitive Analysis of Problems of Comprehension in a Learning of Mathematics. *Educational*

Studies in Mathematics, *61*(1–2), 103–131. https://doi.org/10.1007/s10649-006-0400-z

Herman, T., Rahmi, K., & Utami, N. S. (2022). Student Learning Obstacles in Solving Contextual Mathematical Problems. *AIP Conference Proceedings*, *2659*. https://doi.org/10.1063/5.0113653

Just, A., & Cribbs, J. D. (2020). Focusing on Visual Representations in Mathematics. *Mathematics Teacher: Learning and Teaching PK-12*, *113*(12). https://doi.org/10.5951/mtlt.2019.0139

Levitt, G., & Grubaugh, S. (2023). Teacher-centered or Student-centered Teaching Methods and Stu-dent Outcomes in Secondary Schools: Lecture/Discussion and Pro-ject-based Learning/Inquiry Pros and Cons. *EIKI Journal of Effective Teaching Methods*, *1*(2). https://doi.org/10.59652/jetm.v1i2.16

Malhi, S. K., Kost, C., & Buchanan, L. (2021). Does visualisation help or hinder concrete word processing? *Quarterly Journal of Experimental Psychology*, *74*(2). https://doi.org/10.1177/1747021820956462

Medina-Hidalgo, M. I., Millingalli-Torres, A. E., Guamán-Palate, H. D., & Pozo-Parra, F. F. (2025). El impacto del uso de GeoGebra en el aprendizaje de contenidos matemáticos en la educación básica superior: una revisión bibliográfica. *Multidisciplinary Latin American Journal (MLAJ)*, *3*(3). https://doi.org/10.62131/mlaj-v3-n3-002

Nahum-Shani, I., Shaw, S. D., Carpenter, S. M., Murphy, S. A., & Yoon, C. (2022). Engagement in Digital Interventions. *American Psychologist*, *77*(7). https://doi.org/10.1037/amp0000983

Novaković, A. (2021). The functionality of interactive digital platforms in online teaching and learning. *Nastava i Vaspitanje*, *70*(1). https://doi.org/10.5937/nasvas2101105n

OECD. (2023). *PISA 2022 Assessment and Analytical Framework*. OECD. https://doi.org/10.1787/dfe0bf9c-en

Oktradiksa, A., Mujahidun, M., Hunt, C., & Aufa, M. (2023). A Literacy and Numeracy Model to Enhance the Independent Learning Education for Islamic Elementary School Teachers. *Al Ibtida: Jurnal Pendidikan Guru MI*, *10*(1). https://doi.org/10.24235/al.ibtida.snj.v10i1.13041

Rahmatika, A. (2023). Using Mixed Reality and Adaptive Learning to Enhance Understanding of Algebraic Concepts with the Help of GeoGebra. *Holistic Science*, *3*(3). https://doi.org/10.56495/hs.v3i3.445

Rahmawati, W., Usodo, B., & Fitriana, L. (2021). Mathematical Literacy of Students Who Have High Mathematical Disposition in Solving PISA-Like Mathematics Problems. *Advances in Social Science, Education and Humanities Research*. https://doi.org/DOI:10.2991/assehr.k.211122.039

Rizaldi, M., Pandiangan, P., & Hamdani, M. Y. (2023). Literasi Matematis Mahasiswa Dalam Mengerjakan Masalah Kontekstual Matematika. *Jurnal Pendidikan*, *24*(1). https://doi.org/DOI:10.52850/jpn.v24i1.8886

Sari, Y. M., Arlinwibowo, J., Fatima, G. N., Purwoko, D., & Suprapto. (2024). The potential of virtual representations to help students in learning mathematics. *International Journal of Electrical and Computer Engineering*, *14*(6). https://doi.org/10.11591/ijece.v14i6.pp6411-6422

Sheng, Y., Yu, M., Liu, P., Wang, X., Bai, X., & Zhou, X. (2023). The association between experience-based risky choice and mathematical ability. *PsyCh Journal*, *12*(1). https://doi.org/10.1002/pchj.612

Sofwatun, S., Nurjamil, D., & Dewi, S. V. (2025). Effectiveness of Discovery Learning Assisted by GeoGebra in Enhancing Students' Mathematical

Literacy. *Kognitif: Jurnal Riset HOTS Pendidikan Matematika*, *5*(3). https://doi.org/10.51574/kognitif.v5i3.3491

Suci, S. N., & Reflina, R. (2023). Effect Of Geogebra Towards Students' Independent Learning In Mathematics. *Kalamatika: Jurnal Pendidikan Matematika*, *8*(1). https://doi.org/10.22236/kalamatika.vol8no1.2023pp93-106

Ulul Azmi, S., Sukestiyarno, S., & Rochmad, R. (2023). Students' Mathematical Literacy Ability from van Hiele's Theory in Geometry. *Jurnal Pendidikan MIPA*, *24*(1). https://doi.org/10.23960/jpmipa/v24i1.pp124-134

Uwurukundo, M. S., Maniraho, J. F., & Tusiime, M. (2020). GeoGebra integration and effectiveness in the teaching and learning of mathematics in secondary schools: A review of literature. *African Journal of Educational Studies in Mathematics and Sciences*, *16*(1). https://doi.org/10.4314/ajesms.v16i1.1

Vasile, C. (2022). New "education": learning to cope without learning the abilities. *Journal of Educational Sciences & Psychology*, *12 (74)*(2). https://doi.org/10.51865/jesp.2022.2.01

Zengin, Y. (2019). Development of mathematical connection skills in a dynamic learning environment. *Education and Information Technologies*, *24*(3). https://doi.org/10.1007/s10639-019-09870-x

Zetriuslita, Nofriyandi, & Istikomah, E. (2020). The Increasing Self-Efficacy and Self-Regulated through GeoGebra Based Teaching reviewed from Initial Mathematical Ability (IMA) Level. *International Journal of Instruction*, *14*(1). https://doi.org/10.29333/IJI.2021.14135A

Ziatdinov, R., & Valles, J. R. (2022). Synthesis of Modeling, Visualization, and Programming in GeoGebra as an Effective Approach for Teaching and Learning STEM Topics. In *Mathematics* (Vol. 10, Issue 3). https://doi.org/10.3390/math10030398

Zöchbauer, J., Hohenwarter, M., & Lavicza, Z. (2021). Evaluating GeoGebra Classroom with Usability and User Experience Methods for Further Development. *International Journal for Technology in Mathematics Education*. https://doi.org/DOI:10.1564/tme_v28.3.08