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CHARACTERISTICS OF STUDENTS' MATHEMATICAL PROBLEM-SOLVING STRATEGIES IN PROBLEM-BASED LEARNING THROUGH POLYA'S STAGES

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ABSTRACT

This study aimed to explore the characteristics of students' mathematical problem-solving strategies in Problem-Based Learning through Polya's stages in solving compound interest problems. This study employed a qualitative descriptive case study. The participants consisted of 35 Grade XI students at SMA Negeri 2 Tolitoli, Central Sulawesi. Data were collected through problem-solving tests, interviews, observations, and documentation. Three representative students from high-, moderate-, and low-level problem-solving characteristic profiles were selected purposively for in-depth analysis. Data were analyzed using Miles and Huberman's interactive model. The findings showed that students demonstrated different strategy characteristics across Polya's stages. At the understanding stage, students demonstrated complete identification, partial understanding, and misunderstanding strategies. During the planning stage, conceptual, procedural, and trial-and-error strategies emerged. At the implementation stage, students demonstrated systematic solution, procedural error, and inconsistent-step strategies, while at the reflective stage, students showed solution evaluation, partial verification, and no verification strategies. Students with high-level profiles tended to demonstrate coherent and interconnected strategy patterns, whereas students with low-level profiles demonstrated fragmented and inconsistent processes. The findings further indicated that students' strategies developed as interconnected processes in which difficulties emerging at earlier stages influenced subsequent stages. Reflective activities were identified as the weakest component of students' mathematical problem-solving processes. The findings contribute theoretically to understanding how students' mathematical problem-solving strategies emerge and develop across Polya's stages within Problem-Based Learning environments. Practically, the findings highlight the importance of strengthening reflective activities and supporting interconnected strategic thinking in mathematics learning.

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INTRODUCTION

Mathematical problem-solving ability is recognized as one of the fundamental competencies in mathematics education because it enables students to understand situations, formulate strategies, apply logical reasoning, and communicate mathematical ideas systematically (Lester & Cai, 2015; NCTM, 2000; Schoenfeld, 2016). The National Council of Teachers of Mathematics identifies problem solving as one of the central process standards in mathematics learning alongside reasoning, communication, connections, and representation (Jaenudin et al., 2025; Sugiharti et al., 2026; Syaputra et al., 2025). Problem-solving ability does not merely involve performing calculations but also includes understanding problems, planning solution strategies, applying logical reasoning, and communicating solutions systematically (Jonassen, 2011; Sarmila et al., 2025; Umayrah et al., 2025). In twenty-first-century education, mathematical problem solving is increasingly associated with higher-order thinking skills required to address complex and contextual situations (OECD, 2023; Sugiharti et al., 2026). Therefore, mathematics learning should not merely emphasize procedural computation but should also facilitate students in developing strategic and reflective thinking processes during problem solving.

However, empirical findings indicate that students' mathematical problem-solving ability remains relatively low, particularly in solving contextual and reasoning-based problems (Malik & Mas'ud, 2025; Samosir et al., 2024; Usman et al., 2026). Results from PISA indicate that Indonesian students continue to experience difficulties in mathematical reasoning, modeling, and applying mathematical concepts to real-world situations (Lestari et al., 2021; Linda & Asyura, 2021; OECD, 2023). This condition reflects a gap between the intended goals of mathematics learning and students' actual performance in solving mathematical problems (Schoenfeld, 2016). Similar conditions were also identified in the research setting. Preliminary observations conducted at SMA Negeri 2 Tolitoli, Central Sulawesi, showed that among 35 Grade XI students, only 13 students (37.14%) achieved scores above the minimum mastery criterion (KKM = 75), while 22 students (62.86%) scored below the expected standard. Classroom observations further indicated that students frequently applied formulas directly without adequately understanding problem contexts, identifying relevant information, planning solution strategies, or evaluating the correctness of their answers.

Students' difficulties become particularly evident in mathematical topics closely related to real-life contexts, such as compound interest. Compound interest concepts are associated with financial situations, including savings, loans, investments, and capital growth (Cavalcante et al., 2024; OECD, 2023). However, classroom practice frequently emphasizes formula memorization and procedural calculations rather than conceptual understanding (Ekol & Greenop, 2023; Ncube & Luneta, 2025; Tañola & Lomibao, 2024). As a result, students often experience difficulties in identifying relevant information, modeling contextual situations mathematically, and determining appropriate solution strategies. Such conditions indicate that mathematical problem solving should not only evaluate students' final answers but also examine strategies employed throughout the solution process (Hidayah & Murti, 2025). Analyzing problem-solving strategies provides deeper insights into students' thinking processes when understanding problems, planning solutions, and implementing strategies (Lester & Cai, 2015; Schoenfeld, 2016).

To address these challenges, Problem-Based Learning has been widely recommended as an instructional approach for improving mathematical problem-solving ability (Hmelo-Silver, 2004). PBL places contextual problems as the starting point of learning and actively engages students in investigation, discussion, and collaborative solution development (Putri et al., 2026; Widiastuti et al., 2023). Problem-Based Learning is particularly suitable for mathematical problem solving because students are required to identify problems, explore information, formulate plans, test solution strategies, and evaluate outcomes through authentic situations. These learning activities correspond directly with Polya's stages of understanding the problem, devising a plan, carrying out the plan, and looking back. Therefore, PBL not only improves problem-solving outcomes but also facilitates the emergence of students' strategic behaviors during each stage of the problem-solving process (Hmelo-Silver, 2004; Polya, 1973).

Previous studies reported that PBL improved students' critical thinking and conceptual understanding (Fathurrohman et al., 2024; Uliyandari et al., 2021). Other studies found that PBL promoted critical and creative thinking through active engagement during learning activities (Widiastuti et al., 2023). In addition, studies using Polya's framework identified differences in students' performance across problem-solving stages and reported difficulties during planning and reflective evaluation stages (Fitriani et al., 2022; Yuwono et al., 2025). However, these studies primarily focused on learning outcomes and stage performance rather than critically explaining how students construct and implement problem-solving strategies throughout the learning process (Mustamin et al., 2024). Furthermore, studies specifically analyzing students' problem-solving strategies based on Polya's stages within Problem-Based Learning environments, particularly in financial mathematics topics such as compound interest, remain limited (Auliya et al., 2025; Lutfiani et al., 2026).

These conditions indicate several research gaps. First, limited studies have analyzed students' mathematical problem-solving strategies stage-by-stage using Polya's framework within Problem-Based Learning environments. Second, previous studies rarely focus on financial mathematics contexts such as compound interest, despite their relevance to students' daily financial decision-making involving savings, investments, and loans. Third, previous studies tend to emphasize achievement outcomes while providing a limited understanding of students' strategic characteristics during problem solving. Consequently, little is known about how students' strategies emerge and evolve at each Polya stage within Problem-Based Learning environments, particularly in financial mathematics contexts. Therefore, the novelty of this study lies in integrating Problem-Based Learning and Polya's framework to reveal how students' problem-solving strategies emerge and evolve at each stage of the problem-solving process in compound interest contexts.

Based on the above description, this study aims to explore the characteristics of students' mathematical problem-solving strategies in Problem-Based Learning through Polya's stages in solving compound interest problems. Specifically, this study addresses the following research question: *How do students implement mathematical problem-solving strategies at each stage of Polya's framework within Problem-Based Learning environments when solving compound interest problems?* The findings are expected to contribute theoretically by enriching the understanding of students' mathematical thinking processes and practically by providing instructional implications for strengthening reflective and meaningful mathematical problem solving.

METHOD

This study employed a qualitative descriptive case study design because it aimed to describe in depth the characteristics of students' mathematical problem-solving strategies in Problem-Based Learning based on Polya's stages, rather than to test the effectiveness of a treatment statistically (Creswell, 2014). The case in this study was the implementation of Problem-Based Learning in a Grade XI mathematics classroom on compound interest material. Although students' test responses were scored using an analytic rubric, the scores were used only as supporting descriptive information to support the identification of representative problem-solving characteristic profiles and the selection of interview informants. Therefore, this study was positioned as a qualitative study supported by descriptive quantitative data, not as a mixed-methods study.

The study was conducted at SMA Negeri 2 Tolitoli, Central Sulawesi. The participants consisted of 35 Grade XI students who were involved in the Problem-Based Learning process on compound interest material. All students in the selected class were included as test participants using a total sampling technique because the number of students was limited and the study aimed to obtain a complete description of students' problem-solving strategies in the class.

From the 35 students, three students were selected purposively as key informants for follow-up interviews. The selected informants represented high-, moderate-, and low-level problem-solving characteristic profiles. The selection was based on students' test scores, completeness of written responses, and the clarity of their problem-solving strategy implementation. This selection aimed to obtain deeper insights into how students interpreted problems, formulated strategies, carried out procedures, and evaluated their solutions across Polya's stages.

The research procedure consisted of several sequential stages. First, the researcher prepared the research instruments, including lesson plans, problem-solving tests, scoring rubrics, interview guidelines, and observation sheets developed based on PBL and Polya's stages. Second, the instruments were validated by two mathematics education experts to ensure content suitability, language clarity, and alignment with Polya's problem-solving indicators. The validation yielded a Content Validity Index (CVI) of 0.89, indicating high content validity. Minor revisions included improving contextual wording, refining item alignment with Polya's stages, and clarifying scoring rubric descriptors. After revision, the instruments were considered appropriate for data collection. Third, Problem-Based Learning was implemented in the classroom through activities involving problem orientation, organizing students, guided investigation, developing and presenting solutions, and reflection and evaluation. Fourth, students completed contextual essay tests designed to explore their mathematical problem-solving strategies in compound interest problems. Based on the test results, follow-up interviews were conducted for deeper exploration. Fifth, interviews and classroom observations were conducted to obtain deeper information regarding students' problem-solving strategies. Finally, the collected data were analyzed using Miles and Huberman's interactive model involving data reduction, data display, and conclusion drawing and verification (Miles et al., 2014). The overall research procedure is illustrated in Figure 1.

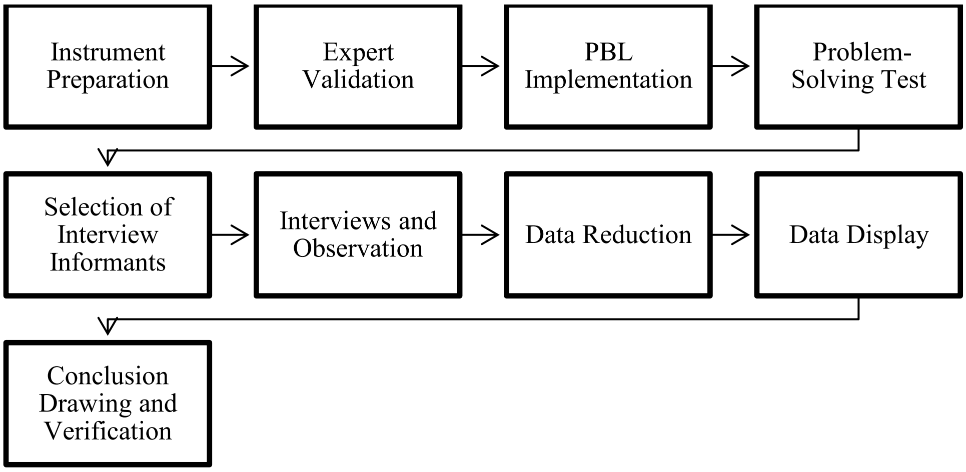

Figure 1. Research Procedure

Data were collected through problem-solving tests, semi-structured interviews, classroom observations, and documentation. The problem-solving test was developed based on Polya's four stages to identify students' written strategies in solving compound interest problems. Semi-structured interviews with selected students were conducted to explore the reasoning underlying their written responses. Classroom observations focused on students' interactions and strategy development during Problem-Based Learning activities, while documentation included written responses, scoring records, and field notes.

To ensure alignment between the research objectives and assessment indicators, a mathematical problem-solving test blueprint was developed based on Polya's four problem-solving stages. The blueprint specifies the indicators assessed, the corresponding stages of problem solving, and the mathematical concepts involved in compound interest problems. The structure of the test blueprint is presented in Table 1.

Table 1. Blueprint of Mathematical Problem-Solving Test

Indicator	Polya's Stage	Item Type	Concept
Identifying known information	Understanding	Contextual essay	Compound interest concept
Determining strategy	Devising a plan	Essay	Compound interest formula
Performing calculations	Carrying out the plan	Computational task	$A = P(1+r)^n$
Reviewing results	Looking back	Interpretation	Meaning of the solution

The following is an example of the contextual problem used in the test:

> "A student deposits Rp2,000,000 in a bank with an annual compound interest rate of 5%. If the money is saved for 3 years, determine the final amount of money received by the student. Explain the known information, determine the strategy used, solve the problem systematically, and check whether the answer is reasonable".

This item was designed to encourage students to demonstrate all stages of Polya's problem-solving process, including understanding the problem, planning a solution, carrying out calculations, and reviewing the result.

Students' responses were assessed using an analytic scoring rubric based on Polya's stages, enabling systematic evaluation of students' performance at each stage of problem solving (Jonassen, 2011).

Table 2. Scoring Rubric Based on Polya's Stages

Stage	Score 3	Score 2	Score 1	Score 0
Understanding	Complete and correct	Partial	Incorrect	No response
Devising a plan	Appropriate strategy	Less appropriate	Unclear	No strategy
Carrying out the plan	Systematic and correct	Minor errors	Inconsistent	No solution
Looking back	Complete evaluation	Partial	Unclear	No evaluation

The scoring results were used only as descriptive information to support the identification of representative problem-solving characteristic profiles rather than to determine students' mathematical abilities. The main analysis, however, focused on the qualitative characteristics of students' strategies at each stage.

Table 3. Interview Guideline Based on Polya's Stages

Polya Stage	Focus	Sample Question
Understanding	Identifying information	What information did you identify from the problem?
Devising a Plan	Planning strategy	Why did you choose this strategy/formula?
Carrying Out	Implementing procedures	How did you perform the calculation process?
Looking Back	Evaluating solutions	Did you check your answer? Why?

The interview guidelines were developed based on Polya's stages to explore students' reasoning processes at each stage of problem solving.

Table 4. Observation Indicators During PBL Activities

Polya Stage	Indicator
Understanding	Identifies relevant information
Devising a plan	Discusses and selects strategies
Carrying out	Applies procedures systematically
Looking back	Reviews and evaluates solutions

Classroom observations were guided by indicators developed from Polya's stages and focused on students' interactions and strategy development during Problem-Based Learning activities.

Data analysis followed Miles and Huberman's interactive model involving data reduction, data display, and conclusion drawing/verification (Miles et al., 2014). During data reduction, students' written responses, interview transcripts, observations, and documentation were organized and coded according to Polya's stages. In the data display stage, strategy patterns were presented through tables and descriptive narratives. Finally, conclusions were drawn and verified through iterative comparison across data sources to identify the characteristics of students' mathematical problem-solving strategies.

Multiple data sources were integrated to strengthen the depth and credibility of qualitative findings. To ensure the trustworthiness of the findings, this study applied credibility, dependability, confirmability, and transferability. Credibility was enhanced through source triangulation by comparing students' written test responses, interview results, classroom observations, and documentation. Cross-interpretation was also conducted by involving another mathematics education expert to review selected students' responses and the researcher's interpretation of their strategies.

Dependability was ensured by applying consistent research procedures, including standardized test administration, interview guidelines, and coding procedures. Confirmability was supported by maintaining documentation of students' written work, scoring sheets, interview notes, and analysis records. Transferability was strengthened by providing a detailed description of the research context, participants, learning procedures, instruments, and analysis process.

RESULT AND DISCUSSION

1. Preliminary Identification of Students' Problem-Solving Characteristics

To support the identification of students' mathematical problem-solving strategy characteristics and the selection of representative interview informants, a contextual problem-solving test consisting of three essay items was administered to 35 Grade XI students at SMA Negeri 2 Tolitoli. The test was developed based on Polya's four stages of problem solving: understanding the problem, devising a plan, carrying out the plan, and looking back. Students' responses were assessed using an analytic scoring rubric to obtain an initial description of students' problem-solving characteristics and to support the selection of representative interview informants.

The initial analysis revealed substantial variation in students' mathematical problem-solving characteristics. Some students demonstrated relatively complete understanding of problems, appropriate planning strategies, systematic implementation of procedures, and organized solution processes. In contrast, other students experienced difficulties in interpreting problem situations and implementing solution procedures consistently. These findings suggest differences not only in students' strategy profiles but also in how they approached and implemented mathematical problem-solving strategies.

The distribution of students' mathematical problem-solving performance is presented in Table 5.

Table 5. Frequency Distribution of Students' Mathematical Problem-Solving Performance

Category	Score Range	Frequency (F)	Percentage (%)
Very Good	86–100	9	25.71
Good	71–85	5	14.29
Fair	56–70	8	22.86
Poor	≤ 55	13	37.14
Total		35	100

As shown in Table 5, the Poor category represented the largest proportion of students (37.14%), indicating that many students still experienced difficulties in systematically implementing mathematical problem-solving procedures. Meanwhile, only 25.71% of students were categorized as Very Good, suggesting that relatively few students demonstrated well-structured problem-solving performance. Overall, the distribution reflects considerable variation in students' approaches and implementation of problem-solving strategies.

The observed variation provided an important basis for subsequent qualitative exploration. Since the purpose of this study was not merely to classify students according to performance levels but to examine the characteristics of their mathematical problem-solving strategies, representative students were purposively selected for in-depth analysis. The selection considered students' test scores, completeness of written responses, and clarity in implementing solution strategies. The selected students represented high, moderate, and low levels of problem-solving characteristics.

The selected representative students are presented in Table 6.

Table 6. Representative Interview Informants

Informant	Performance Level	Score	Selection Criteria
S2	High	97.92	Complete and systematic responses
S12	Moderate	62.50	Partial but interpretable strategies
S28	Low	27.08	Incomplete and inconsistent strategies

These representative students were subsequently analyzed through interviews and classroom observations to explore how mathematical problem-solving strategies emerged and developed across Polya's stages during Problem-Based Learning activities. The qualitative exploration aimed to provide a deeper understanding of how students' strategies evolved across stages and how difficulties emerging at earlier stages influenced subsequent problem-solving processes.

2. Characteristics of Students' Problem-Solving Strategies at the Understanding Stage

The understanding stage represents the initial phase of Polya's problem-solving process in which students identify known information, determine required information, and interpret the problem situation before developing solution strategies. Analysis of students' written responses indicated variations in the strategies employed during this stage. Based on students' responses, three categories of understanding strategies emerged: complete identification, partial understanding, and misunderstanding of the problem.

The distribution of students' strategies at the understanding stage is presented in Table 7.

Table 7. Distribution of Students' Strategies at the Understanding Stage

Strategy Category	Frequency	Percentage (%)	Characteristics
Complete Identification	18	51.43	Students identified known and required information systematically and correctly
Partial Understanding	11	31.43	Students identified only part of the information
Misunderstanding of the Problem	6	17.14	Students incorrectly interpreted or omitted essential information
Total	35	100	

Based on Table 7, more than half of the students (51.43%) demonstrated complete identification strategies by systematically identifying relevant information from the problem context. These students generally listed known values such as principal amount, interest rate, and saving period before proceeding to subsequent stages. Meanwhile, 31.43% of students demonstrated partial understanding, where only some relevant information was identified. A smaller group of students (17.14%) misunderstood the problem and failed to identify important information correctly.

To obtain a deeper understanding of these strategy variations, representative students from high-, moderate-, and low-level profiles were analyzed in greater detail.

Student S2, representing the high-level problem-solving characteristic profile, demonstrated a complete identification strategy. The student systematically identified the principal amount, annual interest rate, and saving period before planning a solution. Classroom observations also showed that S2 actively discussed relevant information during group activities and carefully reviewed problem details before selecting formulas.

An excerpt from the interview illustrates this process:

> "I first looked at the numbers in the problem and wrote what was known and what should be found because I needed them before choosing the formula." (Interview with S2)

The written response of S2 also showed a structured organization of known and unknown information before entering the calculation stage.

In contrast, student S12, representing the moderate-level problem-solving characteristic profile, demonstrated partial understanding. The student identified some numerical information but omitted certain details required for complete problem interpretation.

During the interview, S12 stated:

> "I wrote only the values that I thought were important and directly continued to the formula." (Interview with S12)

Observation data indicated that S12 tended to focus immediately on computational procedures without thoroughly discussing problem conditions.

A different pattern appeared in student S28, representing the low-level problem-solving characteristic profile. This student demonstrated misunderstanding of the problem by incorrectly identifying relevant information and directly attempting calculations without sufficient interpretation.

The interview data revealed:

> "I just tried using the formula because I thought all savings problems use the same way." (Interview with S28)

Classroom observations also showed that S28 rarely revisited problem statements and tended to rely on immediate procedural attempts.

These findings indicate that students demonstrated different characteristics in understanding mathematical problems. Students with high-level profiles tended to employ complete identification strategies characterized by systematic interpretation and careful information selection. Students with moderate-level profiles demonstrated partial understanding by focusing only on selected information, whereas students with low-level profiles frequently relied on assumptions and immediate procedural actions without adequate problem interpretation.

The findings suggest that the understanding stage plays a significant role in shaping subsequent problem-solving processes because difficulties in identifying problem information may influence students' strategy selection and implementation at later stages.

3. Characteristics of Students' Problem-Solving Strategies at the Devising a Plan Stage

The devising a plan stage represents the second phase of Polya's problem-solving process, in which students determine appropriate strategies before carrying out mathematical procedures. At this stage, students are expected to select relevant concepts, formulate solution plans, and identify suitable methods for solving compound interest problems. Analysis of students' written responses and interview data indicated variations in the planning strategies they employed. Based on these findings, three categories of planning strategies emerged: conceptual strategy, procedural strategy, and trial-and-error strategy.

The distribution of students' strategies at the devising a plan stage is presented in Table 8.

Table 8. Distribution of Students' Strategies at the Devising a Plan Stage

Strategy Category	Frequency	Percentage (%)	Characteristics
Conceptual Strategy	14	40.00	Students selected formulas and procedures based on conceptual understanding
Procedural Strategy	13	37.14	Students directly applied formulas without explicit reasoning
Trial-and-Error Strategy	8	22.86	Students attempted various procedures without clear planning
Total	35	100	

Based on Table 8, the conceptual strategy emerged as the most frequently observed category (40.00%). Students applying this strategy generally demonstrated an understanding of relationships among problem information before selecting mathematical procedures. They tended to identify problem characteristics and connect them with compound interest concepts before determining solution methods. Procedural strategy accounted for 37.14% of students, indicating that many students relied directly on formula application without explicitly explaining the rationale behind their choices. Meanwhile, the trial-and-error strategy represented 22.86% of students and reflected difficulties in constructing systematic solution plans.

To obtain a deeper understanding of these planning strategy variations, representative students from high-, moderate-, and low-level profiles were analyzed in greater detail.

Student S2, representing the high-level problem-solving characteristic profile, demonstrated a conceptual strategy. Before performing calculations, the student explained why the compound interest formula was appropriate for solving the problem. The student showed awareness of the relationships among principal amount, interest rate, and time period before selecting procedures. Classroom observations further indicated that S2 actively discussed alternative approaches during group activities and carefully justified the selected solution method.

This reasoning was reflected in the interview:

> "Because the problem involved annual growth and repeated interest, I used the compound interest formula. I checked first whether the information matched the formula requirements." (Interview with S2)

The written response also demonstrated explicit connections between problem information and the selected mathematical procedure.

In contrast, student S12, representing the moderate-level problem-solving characteristic profile, demonstrated a procedural strategy. The student selected a formula immediately without clearly explaining the underlying reasoning behind the decision.

During the interview, S12 stated:

> "I used the formula that I remembered because problems like this usually use the same formula." (Interview with S12)

Observation data indicated that S12 tended to rely on previously learned procedures rather than examining conceptual relationships within the problem context.

Different characteristics emerged in student S28, representing the low-level problem-solving characteristic profile. The student demonstrated a trial-and-error strategy characterized by repeated attempts without a clear solution plan.

Interview data revealed:

> "I was not sure which formula should be used, so I tried different calculations." (Interview with S28)

The written response indicated multiple revisions and inconsistent attempts before arriving at an incomplete solution. Classroom observations also showed that S28 frequently changed approaches and relied heavily on peer discussions during Problem-Based Learning activities.

These findings indicate that students demonstrated different characteristics in planning mathematical problem-solving strategies. Students with high-level profiles tended to employ conceptual strategies characterized by explicit reasoning and meaningful concept selection. Students with moderate-level profiles relied more heavily on procedural strategies through direct formula application, whereas students with low-level profiles frequently demonstrated trial-and-error approaches characterized by uncertainty and inconsistent planning.

The findings suggest that planning strategies play a significant role in shaping subsequent problem-solving processes. Students who established conceptual plans tended to demonstrate more organized implementation patterns in later stages, whereas unclear planning frequently resulted in procedural inconsistency during problem solving.

4. Characteristics of Students' Problem-Solving Strategies at the Carrying Out the Plan Stage

The carrying out the plan stage represents the implementation phase of Polya's problem-solving process in which students execute previously selected strategies and procedures. At this stage, students are expected to apply mathematical operations systematically, perform calculations accurately, and maintain consistency between planned strategies and actual implementation. Analysis of students' written responses and interview data revealed variations in the implementation strategies used at this stage. Based on the findings, three categories of implementation strategies emerged: systematic solution, procedural error, and inconsistent steps.

The distribution of students' strategies in carrying out the plan stage is presented in Table 9.

Table 9. Distribution of Students' Strategies in Carrying Out the Plan Stage

Strategy Category	Frequency	Percentage (%)	Characteristics
Systematic Solution	20	57.14	Students performed procedures sequentially and consistently
Procedural Error	10	28.57	Students followed appropriate procedures but made calculation errors
Inconsistent Steps	5	14.29	Students demonstrated disorganized or changing procedures
Total	35	100	

Based on Table 9, the systematic solution strategy emerged as the most dominant category, accounting for 57.14% of students. Students in this category generally followed solution procedures sequentially and implemented calculations consistently according to their selected strategies. Procedural error represented 28.57% of students, indicating that although students selected relatively appropriate procedures, errors occurred during computational implementation. Meanwhile, a smaller proportion of students (14.29%) demonstrated inconsistent steps characterized by fragmented and changing procedures.

To obtain a deeper understanding of these implementation strategy variations, representative students from high-, moderate-, and low-level profiles were analyzed in greater detail.

Student S2, representing the high-level problem-solving characteristic profile, demonstrated a systematic solution strategy. The student implemented calculations in a structured sequence by correctly substituting values into the compound interest formula and performing procedures without major errors. The written response showed organized mathematical expressions and consistent procedural progression. Classroom observations also indicated that S2 frequently reexamined intermediate calculations and actively monitored procedural accuracy during Problem-Based Learning activities.

The interview data further revealed:

> "After selecting the formula, I substituted the values step by step and checked each calculation before continuing." (Interview with S2)

The response illustrates that S2 implemented the selected strategy carefully and maintained consistency throughout the calculation process.

In contrast, student S12, representing the moderate-level problem-solving characteristic profile, demonstrated procedural error characteristics. Although the student followed an appropriate sequence of procedures, several minor calculation mistakes were identified during implementation.

During the interview, S12 explained:

> "I knew the steps, but I sometimes forgot or miscalculated the numbers." (Interview with S12)

Observation findings indicated that S12 appeared relatively confident during calculations but rarely revisited intermediate computational processes.

Different characteristics emerged in student S28, representing the low-level problem-solving characteristic profile. The student demonstrated inconsistent steps characterized by abrupt procedural changes and fragmented calculations.

Interview data revealed:

> "I changed the calculation because I thought the previous one was wrong." (Interview with S28)

The written response showed several incomplete attempts and inconsistent mathematical procedures. Classroom observations further indicated that S28 frequently paused, repeatedly revised procedures, and relied heavily on peer discussions during Problem-Based Learning activities.

These findings indicate that students demonstrated different characteristics in implementing mathematical procedures. Students with high-level profiles tended to employ systematic solution strategies characterized by consistency and procedural accuracy. Students with moderate-level profiles generally followed appropriate procedures but experienced difficulties in computational precision, whereas students with low-level profiles frequently demonstrated inconsistent implementation patterns accompanied by uncertainty and fragmented procedures.

The findings suggest that carrying out the plan is not solely dependent on computational ability but is also influenced by the quality of planning strategies established in earlier stages. Students who developed stronger conceptual planning tended to demonstrate more organized implementation processes, whereas unclear planning often resulted in procedural inconsistency during problem solving.

5. Characteristics of Students' Problem-Solving Strategies at the Looking Back Stage

The looking back stage represents the final phase of Polya's problem-solving process in which students review, verify, and evaluate the appropriateness of their solutions. At this stage, students are expected to reexamine calculations, assess the reasonableness of answers, and reflect on whether the obtained solution is consistent with the problem context. Analysis of students' written responses and interview data indicated variations in reflective strategies employed during this stage. Based on the findings, three categories of reflective strategies emerged: solution evaluation, partial verification, and no verification. The distribution of students' strategies at the looking back stage is presented in Table 10.

Table 10. Distribution of Students' Strategies at the Looking Back Stage

Strategy Category	Frequency	Percentage (%)	Characteristics
Solution Evaluation	9	25.71	Students systematically reviewed procedures and interpreted solutions
Partial Verification	12	34.29	Students checked only selected procedures or calculations
No Verification	14	40.00	Students provided answers without reviewing or evaluating them
Total	35	100	

Based on Table 10, the largest proportion of students (40.00%) demonstrated no verification strategies. These students tended to terminate the problem-solving process immediately after obtaining numerical answers without checking whether their solutions were reasonable. Partial verification accounted for 34.29% of students and reflected limited attempts to review selected procedures or calculations. Only 25.71% of students demonstrated complete solution evaluation, indicating that reflective thinking had not yet become a consistent component of students' mathematical problem-solving processes.

To obtain a deeper understanding of these reflective strategy variations, representative students from high-, moderate-, and low-level profiles were analyzed in greater detail.

Student S2, representing the high-level problem-solving characteristic profile, demonstrated a solution evaluation strategy. After completing calculations, the student revisited procedural steps and evaluated whether the final answer was reasonable within the context of compound interest problems. Observation findings also indicated that S2 frequently reexamined previous calculations and actively reflected on intermediate results during Problem-Based Learning activities.

The interview data revealed:

> "I checked the calculations again because if the savings amount becomes smaller, it means something is wrong." (Interview with S2)

The response illustrates that S2 not only verified calculations but also interpreted the meaning and reasonableness of the obtained solution.

In contrast, student S12, representing the moderate-level problem-solving characteristic profile, demonstrated partial verification characteristics. The student reviewed certain numerical calculations but did not evaluate whether the final answer was conceptually appropriate.

During the interview, S12 stated:

> "I checked some calculations, but I did not really think about whether the answer made sense." (Interview with S12)

Observation findings suggested that S12's reflective activities focused primarily on computational accuracy rather than interpretation of the solution.

Different characteristics emerged in student S28, representing the low-level problem-solving characteristic profile. The student demonstrated no verification behavior and concluded the problem-solving process immediately after obtaining a numerical result.

Interview data indicated:

> "After I got an answer, I thought I had finished." (Interview with S28)

The written response showed no evidence of reviewing procedures, checking calculations, or interpreting the obtained solution. Classroom observations further indicated that S28 rarely revisited completed work and frequently moved directly to other activities after obtaining an answer during Problem-Based Learning activities.

These findings indicate that students demonstrated different characteristics in reflective engagement during mathematical problem solving. Students with high-level profiles tended to perform complete evaluations by reviewing procedures and interpreting solution meaning. Students with moderate-level profiles conducted limited verification focused mainly on calculations, whereas students with low-level profiles frequently omitted reflective activities entirely.

The findings further suggest that reflective evaluation remained one of the weakest components of students' mathematical problem-solving processes. Many students perceived problem-solving as complete once numerical answers had been obtained, indicating that reflective thinking had not yet become an integral component of students' mathematical problem-solving strategies.

6. Cross-Case Interpretation of Emerging Strategy Patterns

To obtain a broader understanding of how students' mathematical problem-solving strategies emerged and developed across Polya's stages, a cross-case interpretation was conducted using representative students from high-, moderate-, and low-level problem-solving characteristic profiles. This analysis integrated students' written responses, interview findings, and classroom observations to identify both common and contrasting patterns across problem-solving stages.

The comparison of students' strategy characteristics is presented in Table 11.

Table 11. Cross-Case Comparison of Students' Strategy Characteristics Across Polya's Stages

Informant	Understanding	Devising a Plan	Carrying Out	Looking Back
S2 (High)	Complete identification	Conceptual strategy	Systematic solution	Solution evaluation
S12 (Moderate)	Partial understanding	Procedural strategy	Procedural error	Partial verification
S28 (Low)	Misunderstanding	Trial-and-error strategy	Inconsistent steps	No verification

Based on Table 11, distinct strategy patterns emerged across students with different problem-solving characteristic profiles. Student S2 consistently demonstrated structured and reflective strategy characteristics throughout all stages. The student systematically identified relevant information, selected strategies based on conceptual understanding, implemented procedures consistently, and evaluated the obtained solutions. This pattern indicates that students with high-level profiles tended to demonstrate integrated problem-solving processes characterized by coherence across stages.

In contrast, student S12 demonstrated partially structured strategy characteristics. Although the student was able to identify relevant information and apply solution procedures, the problem-solving process relied more heavily on procedural actions than conceptual reasoning. Difficulties became more apparent during the implementation and reflective stages, where minor computational errors and limited verification behaviors were observed.

Different characteristics emerged in student S28, whose strategy patterns appeared fragmented across stages. Difficulties identified during the understanding stage influenced subsequent stages, resulting in trial-and-error planning, inconsistent implementation procedures, and the absence of reflective evaluation. Observation findings further indicated that S28 frequently changed strategies and relied heavily on external support during Problem-Based Learning activities.

These findings indicate that students' problem-solving strategies did not emerge as isolated actions within each stage but developed as interconnected processes. Difficulties occurring at earlier stages appeared to influence students' approaches during subsequent stages. Students demonstrating stronger understanding and conceptual planning tended to develop more systematic implementation patterns and stronger reflective evaluation.

The findings further suggest that students' strategy development followed different trajectories across problem-solving stages. Students with high-level profiles demonstrated relatively stable and coherent strategy patterns, students with moderate-level profiles displayed partially connected strategies, whereas students with low-level profiles demonstrated fragmented and inconsistent strategy development.

Therefore, students' mathematical problem-solving strategies can be understood as evolving processes shaped by interactions among problem interpretation, planning decisions, procedural implementation, and reflective evaluation. Rather than functioning as isolated procedural actions, these strategies emerged progressively and developed across Polya's stages within Problem-Based Learning activities.

The findings of this study revealed that students demonstrated diverse characteristics of mathematical problem-solving strategies across Polya's stages within Problem-Based Learning environments. These strategies did not emerge as isolated actions but developed as interconnected processes involving problem interpretation, planning decisions, procedural implementation, and reflective evaluation. Differences in students' strategies were evident among students with high-, moderate-, and low-level problem-solving characteristic profiles and influenced subsequent problem-solving processes.

At the understanding stage, students demonstrated complete identification, partial understanding, and misunderstanding strategies. Students with stronger understanding tended to identify and organize relevant information systematically before proceeding to subsequent stages. These findings support Polya's view that understanding the problem forms the foundation of successful problem solving because difficulties occurring at the initial stage may affect later stages (Polya, 1973). Similar findings have also been reported in previous studies indicating that students often experience difficulties in identifying relevant information in contextual mathematical situations (Fitriani et al., 2022; Lester & Cai, 2015; Schoenfeld, 2016; Yuwono et al., 2025).

At the devising a plan and carrying out the plan stages, conceptual planning appeared to contribute to more organized implementation patterns. Students who established meaningful conceptual relationships before selecting formulas tended to demonstrate systematic procedures, whereas students relying on procedural or trial-and-error approaches frequently experienced implementation difficulties. These findings support the view that planning and implementation represent interconnected stages rather than separate procedural actions (Jonassen, 2011; Mustamin et al., 2024; Polya, 1973).

The weakest characteristics emerged at the looking back stage. Many students perceived problem solving as complete once numerical answers had been obtained and rarely revisited calculations or evaluated whether solutions were reasonable. These findings suggest that reflective thinking remains insufficiently developed and support previous studies indicating that reflection is often one of the least emphasized aspects of mathematics learning (Hidayah & Murti, 2025; Schoenfeld, 2016).

Cross-case analysis further showed that students' strategies developed differently across Polya's stages. Students with high-level profiles demonstrated relatively coherent and interconnected strategy patterns, whereas students with low-level profiles displayed fragmented processes characterized by misunderstanding, inconsistent planning, and limited reflection. These findings suggest that students' strategies developed progressively and were influenced by preceding stages rather than functioning independently.

From the perspective of Problem-Based Learning, the findings suggest that contextual and collaborative learning activities supported students in constructing and refining problem-solving strategies. However, reflective evaluation remained relatively weak, indicating that Problem-Based Learning alone may not automatically strengthen reflective thinking skills. Therefore, instructional scaffolding appears necessary, particularly during reflective stages.

Overall, this study contributes theoretically by demonstrating that mathematical problem-solving strategies represent evolving and interconnected processes across Polya's stages within Problem-Based Learning environments. Pedagogically, the findings suggest the importance of designing learning environments that strengthen reflective activities and support students in developing interconnected strategic thinking processes.

CONCLUSION

This study explored the characteristics of students' mathematical problem-solving strategies in Problem-Based Learning based on Polya's stages. The findings showed that students demonstrated different strategy characteristics across the stages of understanding the problem, devising a plan, carrying out the plan, and looking back. Students with high-level problem-solving characteristic profiles tended to demonstrate coherent and interconnected strategies, whereas students with low-level profiles showed fragmented and inconsistent problem-solving processes.

The findings further indicated that students' strategies developed as interconnected processes in which difficulties emerging at earlier stages influenced subsequent stages. Reflective activities were identified as the weakest component, as many students rarely reviewed or evaluated their solutions after obtaining answers.

Theoretically, this study contributes to understanding how students' mathematical problem-solving strategies emerge and develop across Polya's stages within Problem-Based Learning environments. The findings suggest that students' strategies should be understood as evolving processes rather than isolated procedural actions. Practically, the findings highlight the importance of strengthening reflective activities and supporting students in developing interconnected strategic thinking during mathematics instruction. Future studies may involve broader educational contexts and different mathematical topics to explore strategy patterns across diverse learning situations.

AUTHOR CONTRIBUTIONS

UY and ST conducted the investigation, collected and organized research data, and contributed to drafting the manuscript. N contributed to methodology development, validation of research instruments, formal analysis, and manuscript

revision. WS supervised the research process, contributed to conceptualization, provided resources, and reviewed and edited the manuscript.

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